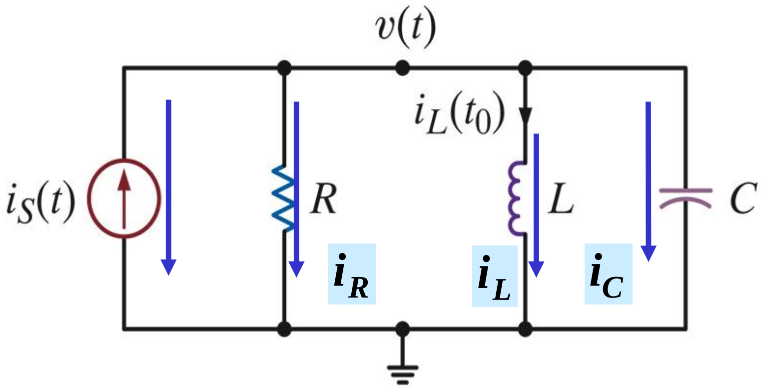
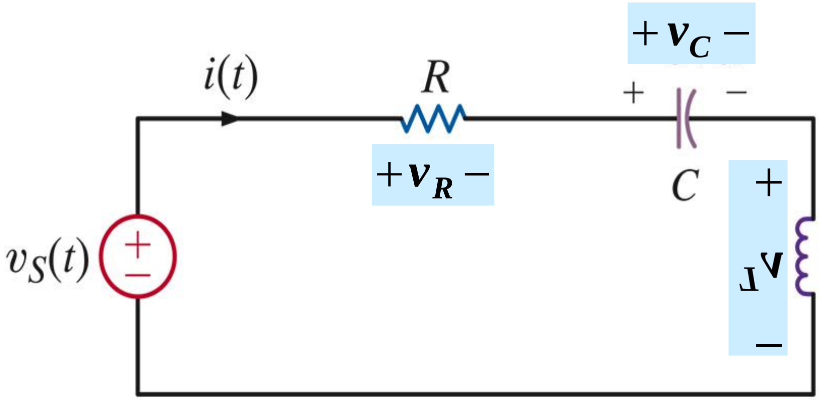
İKİNCİ DERECEDEN DEVRELER

TEMEL DEVRE DENKLEMİ





Tek Düğüm Çiftli Devre: KAK kullan

$$-i_S + i_R + i_L + i_C = 0$$

$$i_R = \frac{v(t)}{R}; \quad i_L = \frac{1}{L}\int_{t_0}^{t} v(x)dx + i_L(t_0); \quad i_C = C\frac{dv}{dt}(t)$$

$$\frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v(x)dx + i_L(t_0) + C\frac{dv}{dt}(t) = i_S$$

Türev alındığında

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = \frac{di_S}{dt}$$

Tek Gözlü Devre: KGK kullan

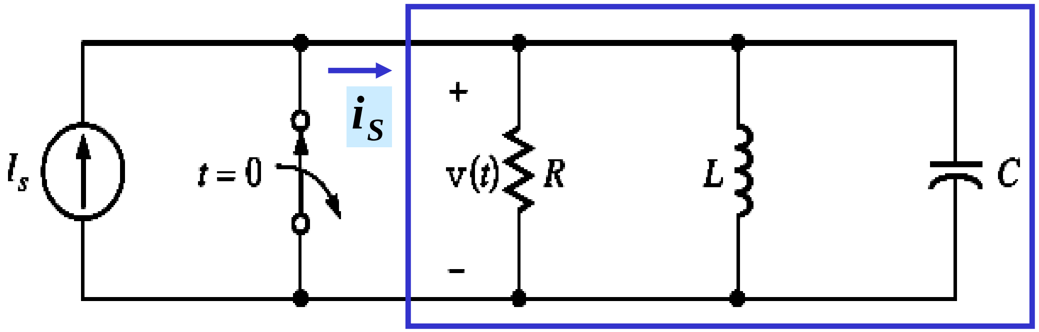
$$-v_S + v_R + v_C + v_L = 0$$

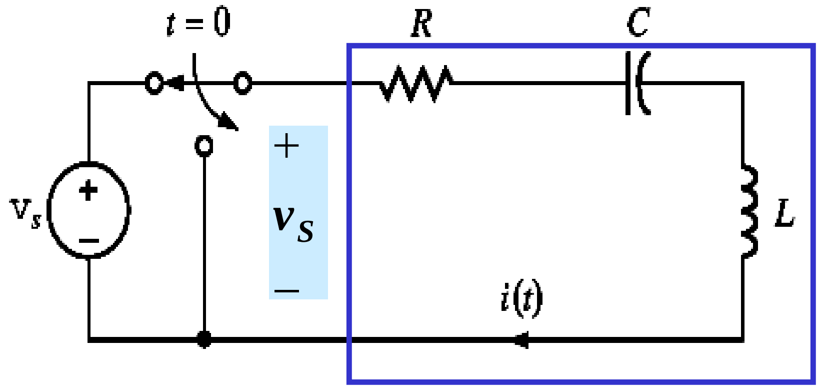
$$v_R = Ri; \; v_C = \frac{1}{C}\int_{t_0}^{t} i(x)dx + v_C(t_0); \; v_L = L\frac{di}{dt}(t)$$

$$Ri + \frac{1}{C}\int_{t_0}^{t} i(x)dx + v_C(t_0) + L\frac{di}{dt}(t) = v_S$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv_S}{dt}$$

ÖRNEK — ***v(t)* ve *i(t)* için Diferansiyel Denklemleri Yazın**





$$i_S(t) = \begin{cases} 0 & t < 0 \\ I_S & t > 0 \end{cases}$$

$$\frac{di_S}{dt}(t) = 0;\ t > 0$$

$$v_S(t) = \begin{cases} V_S & t < 0 \\ 0 & t > 0 \end{cases}$$

$$\frac{dv_S}{dt}(t) = 0;\ t > 0$$

RLC PARALEL DEVRE İÇİN MODEL

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = \frac{di_S}{dt}$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

RLC SERİ DEVRE İÇİN MODEL

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv_S}{dt}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

CEVAP DENKLEMİ

AŞAĞIDAKİ DENKLEMİN ÇÖZÜMÜ

$$\frac{d^2x(t)}{dt^2}+a_1\frac{dx(t)}{dt}+a_0x(t)=f(t)$$

Biliyoruzki: $\boldsymbol{x(t)=x_p(t)+x_c(t)}$

$\boldsymbol{x_p}$ özel çözüm (zorlanmiş)

$\boldsymbol{x_c}$ tamamlayici çözüm (dogal)

Doğal (Tamamlayıcı) çözüm:

$$\frac{d^2x_c(t)}{dt^2}+a_1\frac{dx_c(t)}{dt}+a_0x_c(t)=0$$

Zorlanmış çözüm:

Eğer zorlama fonksiyonu bir sabitse; $f(t)=A$

$$\frac{d^2x_p}{dt^2}=0,\quad \frac{dx_p}{dt}=0,\Rightarrow a_0x_p=A,\Rightarrow\ x_p=\frac{A}{a_0}$$

Herhangi bir zorlama fonksiyonu $f(t)=A$ ise;

Tam çözüm : $x(t)=\dfrac{A}{a_0}+x_c(t)$

DOĞAL (HOMOJEN) DENKLEM

$$\frac{d^2x(t)}{dt^2} + a_1\frac{dx(t)}{dt} + a_0x(t) = 0$$

NORMALIZE EDILMIŞ FORMU

$$\frac{d^2x(t)}{dt^2} + 2\varsigma\omega_n\frac{dx(t)}{dt} + \omega_n^2x(t) = 0$$

ω_n (sönümsüz) dogal frekans

ς sönüm orani

KARAKTERISTIK DENKLEM

$$s^2 + 2\varsigma\omega_n s + \omega_n^2 = 0$$

$$a_0 = \omega_n^2 \Rightarrow \omega_n = \sqrt{a_0}$$

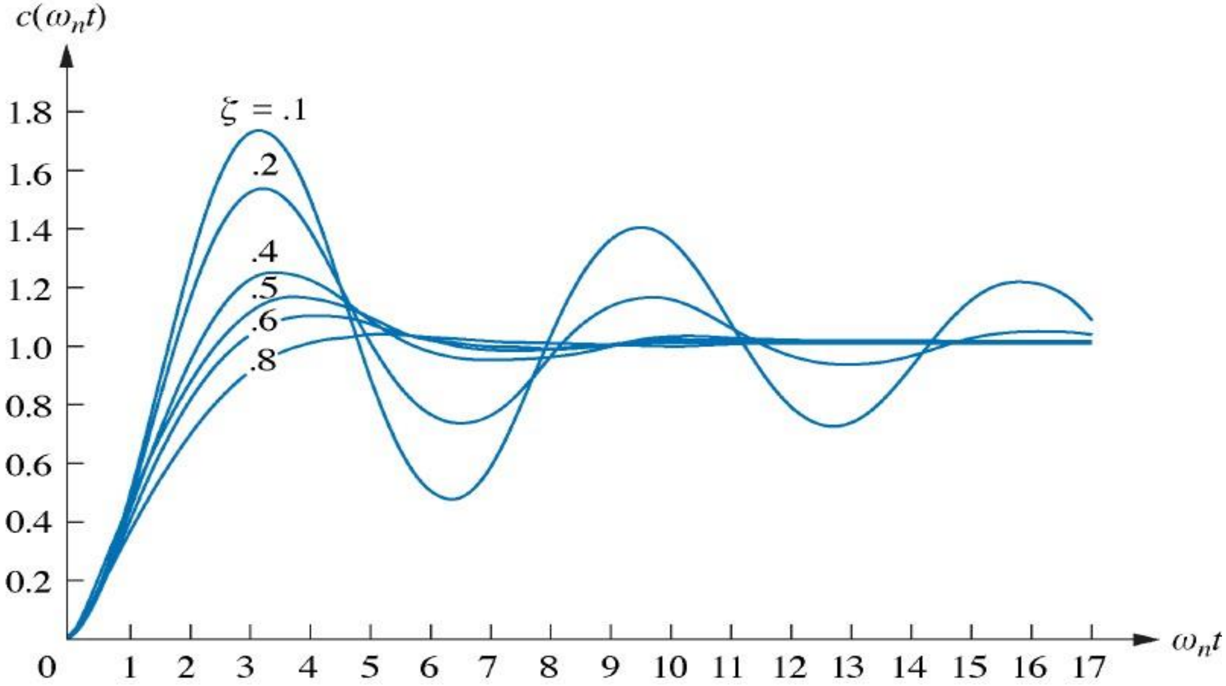
$$a_1 = 2\varsigma\omega_n \Rightarrow \varsigma = \frac{a_1}{2\sqrt{a_0}}$$

İkinci Dereceden Sistemler

İkinci dereceden sistemlerin cevabını iki parametre kullanarak karakterize edebiliriz: ω_n ve ζ

Doğal Frekans, ω_n: Sönümsüz osilasyon frekansıdır. Örneğin, direnci kısa devre yapılmış bir RLC devresinin veya sönümleyicisiz mekanik bir sistemin doğal frekansı. Sönümsüz bir sistem doğal frekans ile tanımlanmaktadır.

Sönümleme Oranı, ζ: Sönümleme miktarını ölçer. Az sönümlü sistemler için ζ sönümleme oranı [0, 1] aralığında bulunur:



RLC SERİ DEVRE İÇİN MODEL

$$\boldsymbol{L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=\frac{dv_S}{dt}}$$

1- Karakteristik denklemi elde etmek için giriş sıfırlanır. (homojen denklem)

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=0$$

2- En büyük türev sabitini bir yapmak için denklem tekrar düzenlenir.

$$\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i=0$$

3- Standart cevap ile karşılaştırılır.

$$\ddot{y}+2\zeta\omega_n\dot{y}+\omega_n^2y=0 \Rightarrow \frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i=0$$

$$\omega_n=\sqrt{\frac{1}{LC}}$$

$$2\varsigma\omega_n=\frac{R}{L}$$

$$\varsigma=\frac{a_1}{2\sqrt{a_0}}=\frac{\frac{R}{L}}{2\sqrt{\frac{1}{LC}}}$$

ÖRNEK

KARAKTERİSTİK DENKLEMİ, SÖNÜM ORANINI VE DOĞAL FREKANSI BELİRLEYİNİZ

$$4\frac{d^2x(t)}{dt^2}+8\frac{dx(t)}{dt}+16x(t)=0$$

İKİNCİ DERECEDEN TÜREVİN KATSAYISI BİR OLMALIDIR.

$$\frac{d^2\boldsymbol{x}}{\boldsymbol{dt}^2}(\boldsymbol{t})+\boxed{2\frac{\boldsymbol{dx}}{\boldsymbol{dt}}(\boldsymbol{t})}+\boxed{4\boldsymbol{x}(\boldsymbol{t})}=0$$

KARAKTERISTIK DENKLEM

$$\boldsymbol{s}^2+\boxed{2\boldsymbol{s}}+\boxed{4}=0$$

SÖNÜM ORANI, DOĞAL FREKANS

$$\frac{d^2\boldsymbol{x}}{\boldsymbol{dt}^2}(\boldsymbol{t})+2\frac{\boldsymbol{dx}}{\boldsymbol{dt}}(\boldsymbol{t})+4\boldsymbol{x}(\boldsymbol{t})=0$$

$$\boxed{2\varsigma\omega_n} \qquad \omega_n^2 \Rightarrow \omega_n=2$$

$$\Downarrow$$

$$\varsigma=0.5$$

DOĞAL (HOMOJEN) DENKLEMİN ANALİZİ

NORMALIZE EDILMIŞ FORMU

$$\frac{d^2x(t)}{dt^2} + 2\varsigma\omega_n \frac{dx(t)}{dt} + \omega_n^2 x(t) = 0$$

Eger

$\boldsymbol{s}^2 + 2\varsigma\omega_{\boldsymbol{n}}\boldsymbol{s} + \omega_{\boldsymbol{n}}^2 = 0$ ise,

çözüm $\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{K}\boldsymbol{e}^{\boldsymbol{st}}$ dir

Eğer s, *karakteristik denklemin* çözümü ise

$$\text{ISPAT}: \frac{dx(t)}{dt} = sKe^{st}; \quad \frac{d^2x}{dt^2} = s^2Ke^{st}$$

$$\frac{d^2x(t)}{dt^2} + 2\varsigma\omega_n \frac{dx(t)}{dt} + \omega_n^2 x(t) = (s^2 + 2\varsigma\omega_n s + \omega_n^2)Ke^{st}$$

KARAKTERISTIK DENKLEM

$$\boldsymbol{s}^2 + 2\varsigma\omega_{\boldsymbol{n}}\boldsymbol{s} + \omega_{\boldsymbol{n}}^2 = 0$$

$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

(sistemin modeli

DURUM 1: $\varsigma > 1$ (gerçek ve ayrı kökler) **Aşırı Sönümlü**

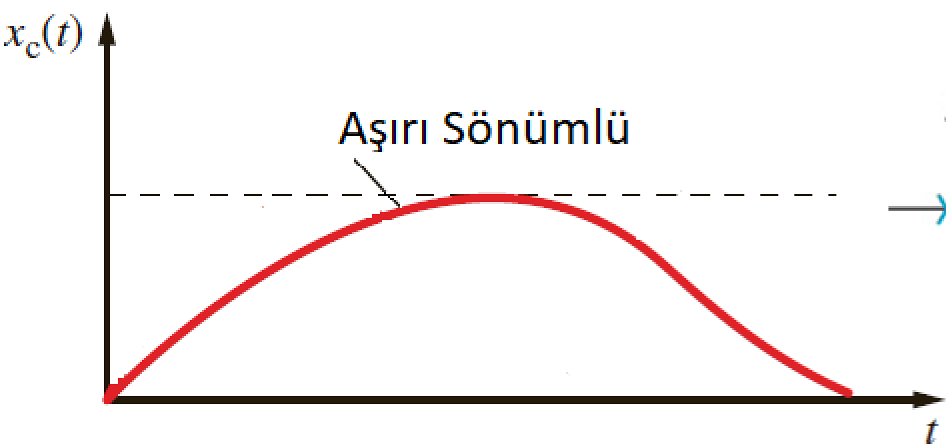
$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

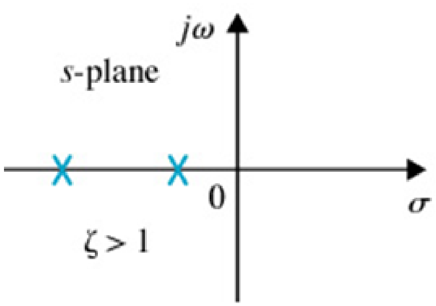
$$s_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

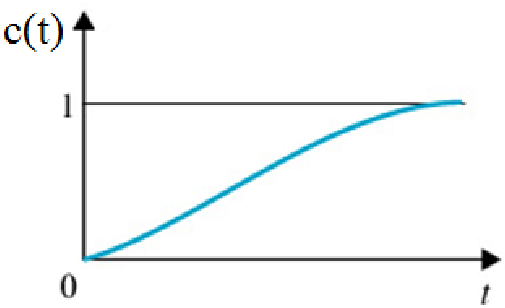
$$s_1 = -\varsigma\omega_n + \omega_n\sqrt{\varsigma^2 - 1}$$

$$s_2 = -\varsigma\omega_n - \omega_n\sqrt{\varsigma^2 - 1}$$

$$x(t) = K_1 e^{-(\varsigma\omega_n - \omega_n\sqrt{\varsigma^2 - 1})t} + K_2 e^{-(\varsigma\omega_n + \omega_n\sqrt{\varsigma^2 - 1})t}$$







DURUM 2: $\varsigma < 1$ (karmasik kökler) **Az Sönümlü**

$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$s_{1,2} = -\varsigma\omega_n \pm j\omega_n\sqrt{1-\varsigma^2}$$
$$s_{1,2} = -\sigma \pm j\omega_d$$

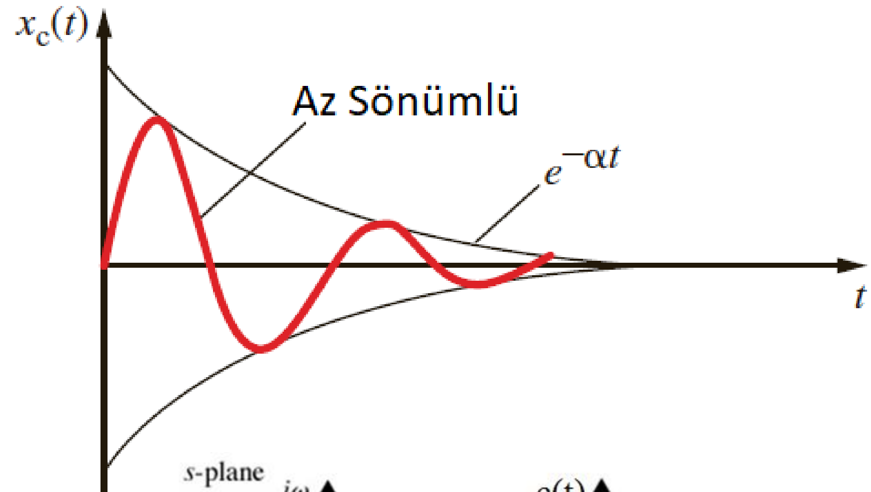
$$s_1 = -\varsigma\omega_n + j\omega_n\sqrt{1-\varsigma^2} = -\sigma + j\omega_d$$
$$s_2 = -\varsigma\omega_n - j\omega_n\sqrt{1-\varsigma^2} = -\sigma - j\omega_d$$

$x(t)$ gerçek $\Rightarrow K_2 = K_1^*$

$\omega_d =$ sönümlü osilosyon frekansi

σ = sönüm katsayisi

$$x(t) = e^{-\sigma t}\left(A_1 \cos\omega_d t + A_2 \sin\omega_d t\right)$$

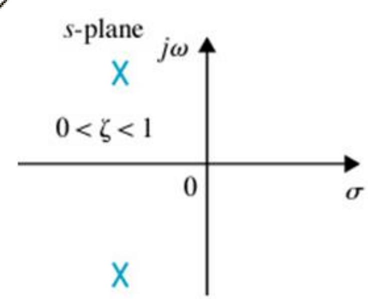


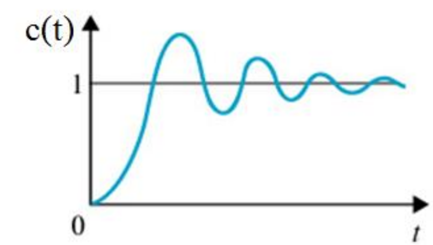
IP UCU: $e^{st} = e^{-(\varsigma\omega_n \pm j\omega_d)t} = e^{-\varsigma\omega_n t} e^{\mp j\omega_d t}$

$$e^{\mp j\omega_d t} = \cos\omega_d t \mp j\sin\omega_d t$$

VARSAYIN $K_1 = (A_1 + jA_2)/2$

$$\left.\begin{matrix} K_2 = K_1^* \\ s = -\sigma \pm j\omega_d \end{matrix}\right\} \Rightarrow x(t) = 2\,\mathrm{Re}\left[K_1 e^{-(\sigma + j\omega_d)t}\right]$$





DOĞAL (HOMOJEN) DENKLEMİN ANALİZi - devamı

DURUM 3: $\varsigma = 1$ (gerçek ve katli kökler) **Kritik Sönümlü**

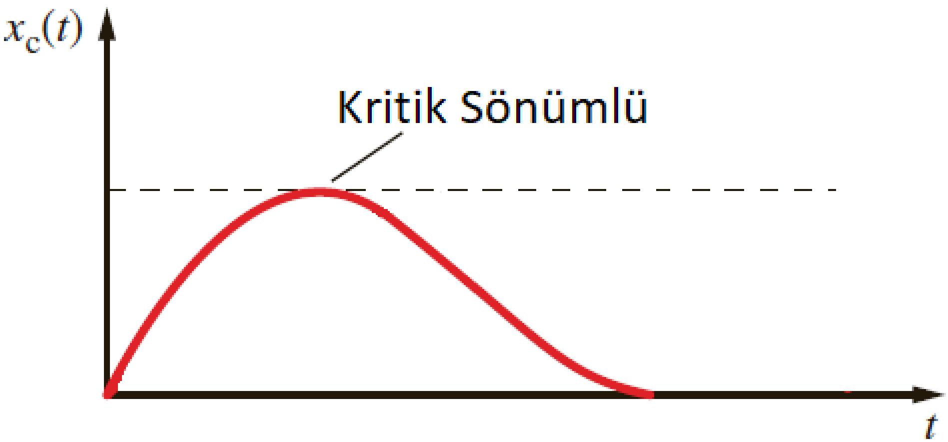
$$x(t) = e^{-\varsigma\omega_n t}\left(B_1 + B_2 t\right)$$

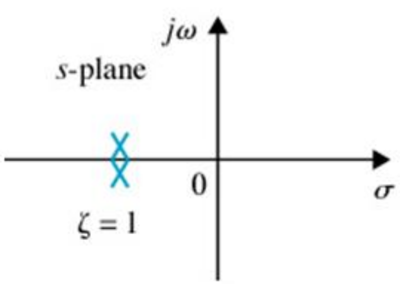
$\boldsymbol{s} = -\varsigma\omega_{\boldsymbol{n}}$ $\boldsymbol{s}_1 = \boldsymbol{s}_2 = -\varsigma\omega_{\boldsymbol{n}}$

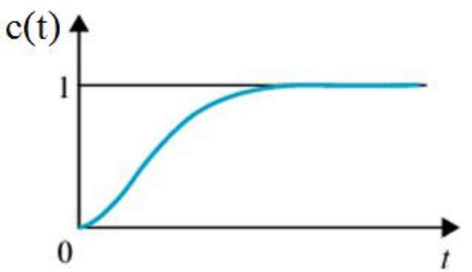
IP UCU: Eger

$(\boldsymbol{s}^2 + 2\varsigma\omega_{\boldsymbol{n}}\boldsymbol{s} + \omega_{\boldsymbol{n}}^2 = 0)$ ***VE*** $(2\boldsymbol{s} + 2\varsigma\omega_{\boldsymbol{n}} = 0)$ ise,

$\boldsymbol{te}^{st}$ çözümdür







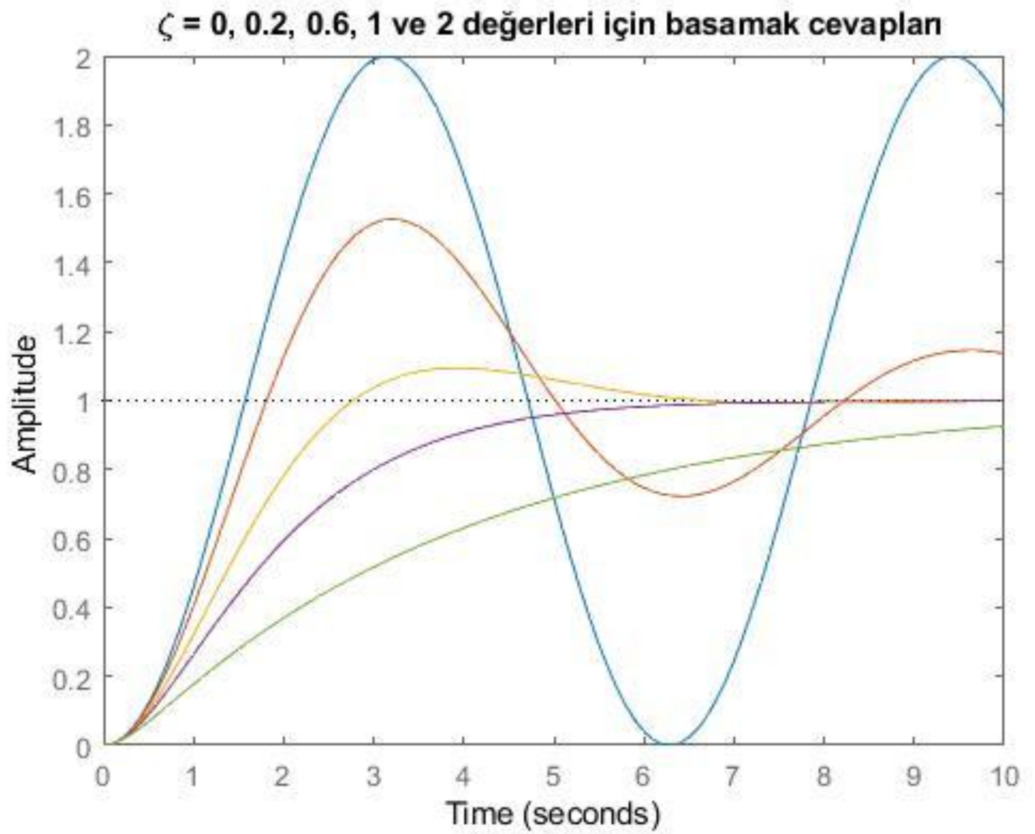
İkinci Dereceden Sistemler

Farklı Sönüm oranları için basamak cevapları

```
t = [0:0.1:10];
pay = [1];
zeta1 = 0.0; payda1 = [1 2*zeta1 1];
zeta2 = 0.2; payda2 = [1 2*zeta2 1];
zeta3 = 0.6; payda3 = [1 2*zeta3 1];
zeta4 = 1.0; payda4 = [1 2*zeta4 1];
zeta5 = 2.0; payda5 = [1 2*zeta5 1];

step(pay,payda1,t); hold on;
step(pay,payda2,t); hold on;
step(pay,payda3,t); hold on;
step(pay,payda4,t); hold on;
step(pay,payda5,t)

title('\zeta = 0, 0.2, 0.6, 1 ve 2
değerleri için basamak cevapları')
```



İKİNCİ DERECEDEN DEVRELER

PROBLEM-SOLVING STRATEGY

STEP 1. Write the differential equation that describes the circuit.

STEP 2. Derive the characteristic equation, which can be written in the form $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$, where ζ is the damping ratio and ω_0 is the undamped natural frequency.

STEP 3. The two roots of the characteristic equation will determine the type of response. If the roots are real and unequal (i.e., $\zeta > 1$), the network response is overdamped. If the roots are real and equal (i.e., $\zeta = 1$), the network response is critically damped. If the roots are complex (i.e., $\zeta < 1$), the network response is underdamped.

STEP 4. The damping condition and corresponding response for the aforementioned three cases outlined are as follows:

Overdamped: $x(t) = K_1 e^{-(\zeta\omega_0 - \omega_0\sqrt{\zeta^2-1})t} + K_2 e^{-(\zeta\omega_0 + \omega_0\sqrt{\zeta^2-1})t}$

Critically damped: $x(t) = B_1 e^{-\zeta\omega_0 t} + B_2 t e^{-\zeta\omega_0 t}$

Underdamped: $x(t) = e^{-\sigma t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$, where $\sigma = \zeta\omega_0$, and

$\omega_d = \omega_0\sqrt{1 - \zeta^2}$

STEP 5. Two initial conditions, either given or derived, are required to obtain the two unknown coefficients in the response equation.

ÖRNEK ÇÖZÜMÜN GENEL BİÇİMİNİ BELİRLEYİN

$$\frac{d^2x}{dt^2}(t) + 4\frac{dx}{dt}(t) + 4x(t) = 0$$

KARAKTERISTIK DENKLEM

$$s^2 + 4s + 4 = 0$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2$$

$$2\varsigma\omega_n = 4 \Rightarrow \varsigma = 1$$

$$s^2 + 4s + 4 = 0 \Rightarrow (s+2)^2 = 0$$

Kökler gerçek ve eşit

Bu kritik sönümlü bir sistemdir (DURUM 3)

$$x(t) = e^{st}(B_1 + B_2 t)$$

$$x(t) = e^{-2t}(B_1 + B_2 t)$$

ÖRNEK ÇÖZÜMÜN GENEL BİÇİMİNİ BELİRLEYİN

$$4\frac{d^2x}{dt^2}(t)+8\frac{dx}{dt}(t)+16x(t)=0$$

İkinci dereceden türev katsayısı ile bölün

$$\frac{d^2x}{dt^2}(t)+2\frac{dx}{dt}(t)+4x(t)=0$$

$$\omega_n^2=4\Rightarrow\omega_n=2$$

$$2\varsigma\omega_n=2\Rightarrow\varsigma=0.5$$

KARAKTERISTIK DENKLEM

$$s^2+2s+4=0$$

$$s^2+2s+4=(s+1)^2+3=0\Rightarrow s=-1\pm j\boxed{\sqrt{3}}$$

ω_d

Kökler karmaşık eşleniktir

Bu az sönümlü bir sistemdir (DURUM 2)

$$\sigma=\varsigma\omega_n=1;\quad \omega_d=\omega_n\sqrt{1-\varsigma^2}=2\sqrt{1-0.25}=\sqrt{3}$$

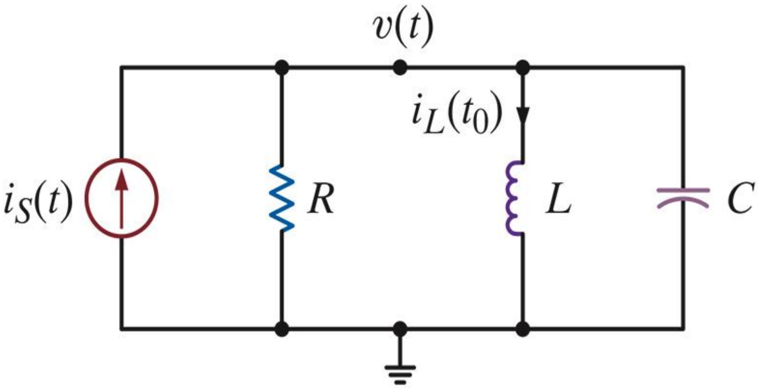
$$x(t)=e^{-\sigma t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$

$$x(t)=e^{-t}\left(A_1\cos\sqrt{3}t+A_2\sin\sqrt{3}t\right)$$

ÖRNEK **Çözümün biçimini belirleyin**

PARALEL *RLC* DEVRESI

$R = 1\Omega, L = 2H, C = 2F$



HOMOJEN DENKLEM

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$2\frac{d^2v}{dt^2} + \frac{dv}{dt} + \frac{v}{2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{2}\frac{dv}{dt} + \frac{v}{4} = 0$$

$$s^2 + \frac{1}{2}s + \frac{1}{4} = (s + \frac{1}{4})^2 + \frac{3}{16} = 0$$

$$\omega_n = \frac{1}{2}; \varsigma\omega_n = \frac{1}{4} \Rightarrow \varsigma = \frac{1}{2}$$

$$\sigma = \frac{1}{4}$$

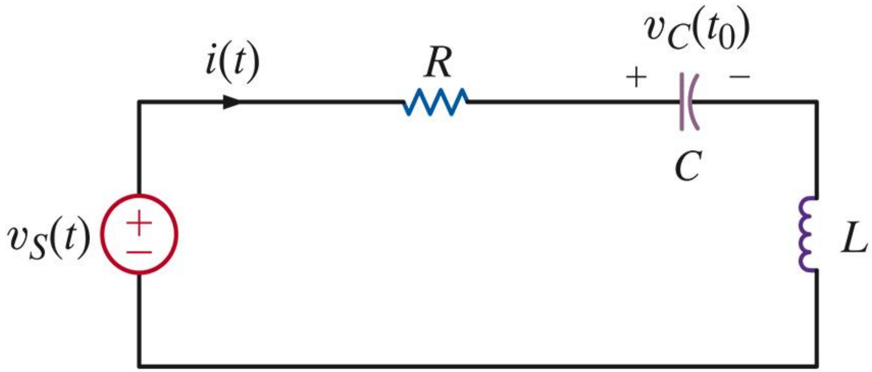
$$\omega_d = \omega_n\sqrt{1-\varsigma^2} = \frac{1}{2}\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{4}$$

$$v_c(t) = e^{-\frac{t}{4}}\left(A_1\cos\frac{\sqrt{3}}{4}t + A_2\sin\frac{\sqrt{3}}{4}t\right)$$

ÖRNEK **Verilen kapasitans değerleri için sistem cevaplarını sınıflandırın**

SERI *RLC* DEVRESI

$R = 2\Omega; L = 1H; C = 0.5F, C = 1F, C = 2F$



HOMOJEN DENKLEM

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 : / L \quad \& \text{degerler yerine yazildigin da}$$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + \frac{i}{C} = 0$$

$$s^2 + 2s + \frac{1}{C} = 0$$

$$\omega_n = \frac{1}{\sqrt{C}}; \; 2\varsigma\omega_n = 2 \Rightarrow \varsigma = \sqrt{C}$$

C=0.5	az sönümlü
C=1.0	kritik sönümlü
C=2.0	aşırı sönümlü

$$\text{diskriminant} = 4 - \frac{4}{C}$$

DEVRE CEVABI

Aşırı Sönümlü

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2x}{dt^2}(t) + 2\varsigma\omega_n \frac{dx}{dt}(t) + \omega_n^2 x(t) = A$$

Biliyoruzki: $x(t) = x_p(t) + x_c(t)$

x_p özel çözüm (zorlanmiş)

x_c tamamlayici çözüm (dogal)

$$x_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x_p(t) = \frac{A}{\omega_n^2}$$

$$x(t) = \frac{A}{\omega_n^2} + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x(0+) - \frac{A}{\omega_n^2} = K_1 + K_2$$

$$\frac{dx}{dt}(0+) = s_1 K_1 + s_2 K_2$$

DEVRE CEVABI

Az Sönümlü

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2x}{dt^2}(t)+2\varsigma\omega_n\frac{dx}{dt}(t)+\omega_n^2x(t)=A$$

Biliyoruzki: $x(t)=x_p(t)+x_c(t)$

x_p özel çözüm (zorlanmiş)

x_c tamamlayici çözüm (dogal)

$$x_c(t)=e^{-\varsigma\omega_n t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$

$$x_p(t)=\frac{A}{\omega_n^2}$$

$$x(t)=\frac{A}{\omega_n^2}+e^{-\varsigma\omega_n t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$

$$x(0+)-\frac{A}{\omega_n^2}=A_1$$

$$\frac{dx}{dt}(0+)=-\varsigma\omega_n A_1+\omega_d A_2$$

DEVRE CEVABI

Kritik Sönümlü

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2x}{dt^2}(t)+2\varsigma\omega_n\frac{dx}{dt}(t)+\omega_n^2x(t)=A$$

Biliyoruzki: $x(t)=x_p(t)+x_c(t)$

x_p özel çözüm (zorlanmiş)

x_c tamamlayici çözüm (dogal)

$$x_c(t)=e^{-\varsigma\omega_n t}\left(B_1+B_2t\right)$$

$$x_p(t)=\frac{A}{\omega_n^2}$$

$$x(t)=\frac{A}{\omega_n^2}+e^{-\varsigma\omega_n t}\left(B_1+B_2t\right)$$

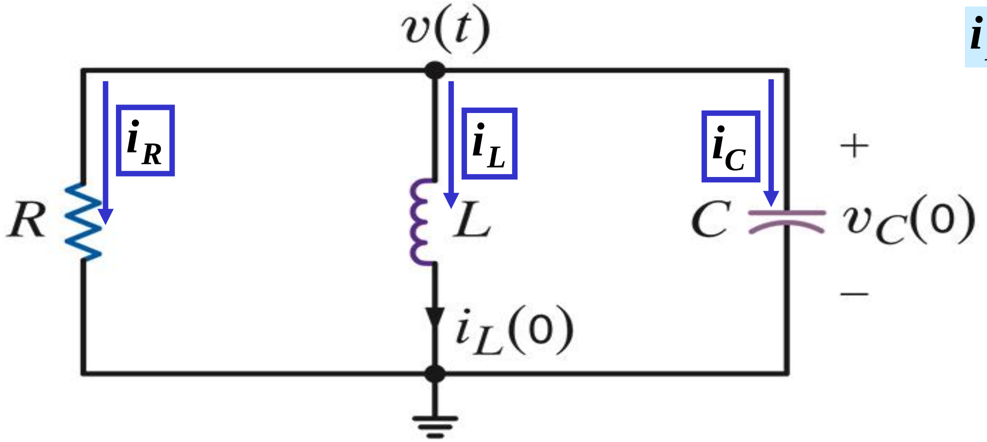
$$x(0+)-\frac{A}{\omega_n^2}=B_1$$

$$\frac{dx}{dt}(0+)=-\varsigma\omega_nB_1+B_2$$

ÖRNEK **v(t) GERİLİM CEVABINI BELİRLEYİN**

$R = 2\Omega, L = 5H, C = 1/5\, F$

$i_L(0) = -1A,\ v_C(0) = 4V$



ADIM 1: MODELİ ELDE EDİN

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int_0^t v(x)dx + i_L(0) + C\frac{dv}{dt} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

ADIM 2: KARAKTERİSTİK DENKLEM

KARAKTERISTIK DENKLEM

$s^2 + 2.5s + 1 = 0$ $\Rightarrow \omega_n = 1;\ \varsigma = 1.25$

ADIM 3: KÖKLERİ BULUN

$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN

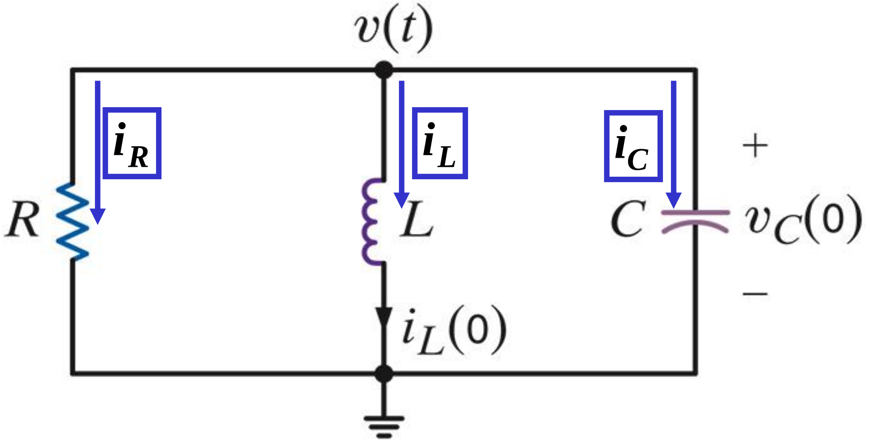
$$v(t) = K_1 e^{-2t} + K_2 e^{-0.5t}$$

ÖRNEK -devam

v(t) GERİLİM CEVABINI BELİRLEYİN

$$R = 2\Omega, L = 5H, C = \tfrac{1}{5}F$$

$$i_L(0) = -1A,\ v_C(0) = 4V$$



ADIM 5: SABİTLERİ BULUN

Sabitleri belirlemek için ihtiyaç duyduklarımız;

$$v(0+);\ \frac{dv}{dt}(0+)$$

$$v(0+) = v_C(0+) = v_C(0) = 4V$$

Eger verilmemisse $v_C(0)$ ve $i_L(0)$ bulunur

t=(0+)' DA DEVREYİ ANALİZ EDİN

$t = (0+)$'DA KAK UYGULA

$$\frac{v_C(0+)}{R} + i_L(0+) + C\frac{dv}{dt}(0+) = 0$$

$$\frac{4}{2} - 1 + \frac{1}{5}\frac{dv}{dt}(0+) = 0$$

$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} + \frac{1}{(1/5)} = -5$$

$$\left.\begin{aligned} K_1 + K_2 &= 4 \\ -2K_1 - 0.5K_2 &= -5 \end{aligned}\right\} \Rightarrow K_1 = 2;\ K_2 = 2$$

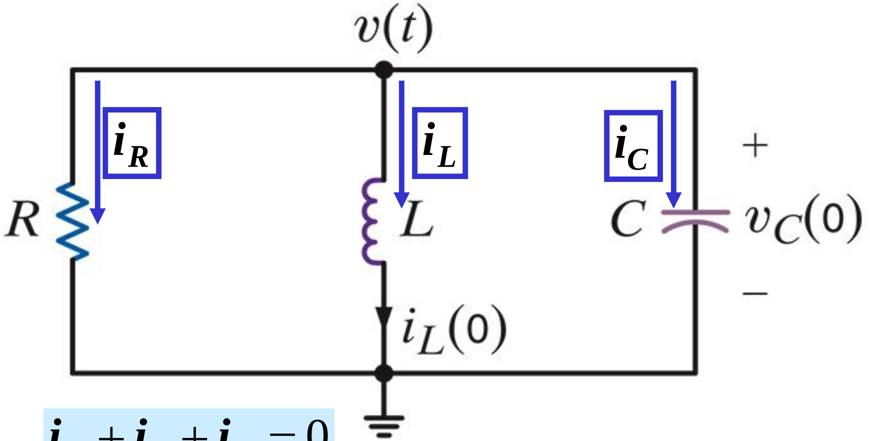
$$v(t) = 2e^{-2t} + 2e^{-0.5t};\ t > 0$$

ÖRNEK -özet

$v(t)$ GERİLİM CEVABINI BELİRLEYİN

$R = 2\Omega, L = 5H, C = \frac{1}{5}F$

$i_L(0) = -1A, v_C(0) = 4V$



$i_R + i_L + i_C = 0$

$$\frac{v}{R} + \frac{1}{L}\int_0^t v(x)dx + i_L(0) + C\frac{dv}{dt} = 0$$

ADIM 1 MODEL

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

KARAKTERISTIK DENKLEM

ADIM 2

$s^2 + 2.5s + 1 = 0 \Rightarrow \omega_n = 1;\ \varsigma = 1.5$

$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

ADIM 3 KÖKLER

$v(t) = K_1e^{-2t} + K_2e^{-0.5t}$

ADIM 4 ÇÖZÜMÜN BİÇİMİ

Sabitleri belirlemek için ihtiyaç duyduklarımız

$v(0+);\ \frac{dv}{dt}(0+)$

Verilmemisse $v_C(0)$ ve $i_L(0)$ bulunur

$v(0+) = v_C(0+) = v_C(0) = 4V$

$t = (0+)$'DA KAK UYGULA

$$\frac{v_C(0+)}{R} + i_L(0+) + C\frac{dv}{dt}(0+) = 0$$

$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} - \frac{(-1)}{(1/5)} = -5$$

t=0+'DA ANALİZ ET

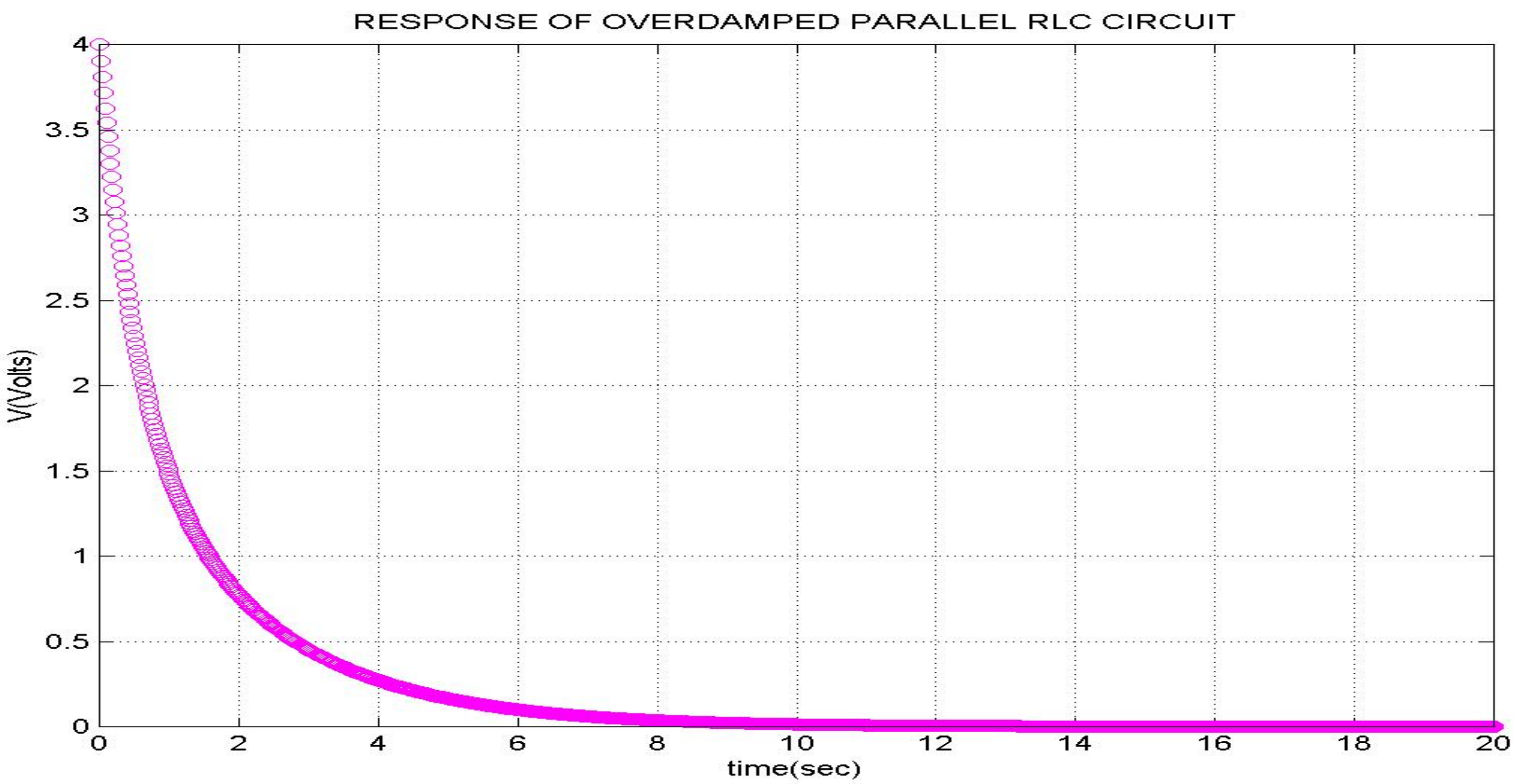
ADIM 5: SABİTLERİ BULUN

$$\left.\begin{aligned} K_1 + K_2 &= 4 \\ -2K_1 - 0.5K_2 &= -5 \end{aligned}\right\} \Rightarrow K_1 = 2;\ K_2 = 2$$

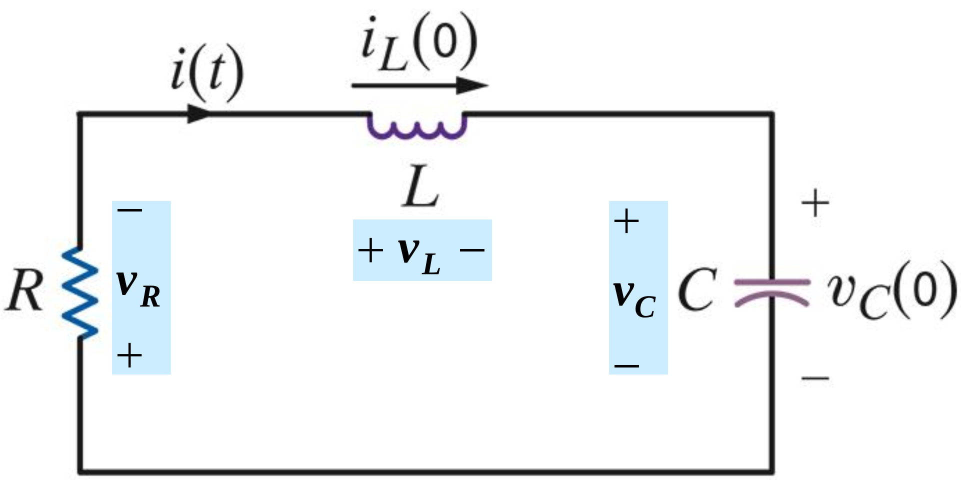
$v(t) = 2e^{-2t} + 2e^{-0.5t};\ t > 0$

CEVABI GÖRSELLEŞTİRMEK İÇİN MATLAB KULLANMA

```
%script6p7.m
%plots the response in Example 6.7
%v(t)=2exp(-2t)+2exp(-0.5t); t>0
t=linspace(0,20,1000);
v=2*exp(-2*t)+2*exp(-0.5*t);
plot(t,v,'mo'), grid, xlabel('time(sec)'), ylabel('V(Volts)')
title('RESPONSE OF OVERDAMPED PARALLEL RLC CIRCUIT')
```



ÖRNEK **$i(t)$ AKIM CEVABINI BELİRLEYİN**



$$v_R + v_L + v_C = 0$$

$R = 6\Omega, L = 1H, C = 0.04F$

$i_L(0) = 4A;\ v_C(0) = -4V$

ADIM 1: MODELİ ELDE EDİN

ADIM 2: KARAKTERİSTİK DENKLEM

ADIM 3: KÖKLERİ BULUN

ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN

ADIM 5: SABİTLERİ BULUN

$$Ri(t) + L\frac{di}{dt}(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0) = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt}(t) + \frac{1}{LC}i(t) = 0$$

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt}(t) + 25i(t) = 0$$

model

Karakteristik Denklem:

$$s^2 + 6s + 25 = 0$$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5$$

$$2\varsigma\omega_n = 6 \Rightarrow \varsigma = 0.6$$

Kökler: $$s = \frac{-6 \pm \sqrt{36 - 100}}{2} = -3 \pm j\boxed{4}\ \omega_d$$

Çözümün biçimi:

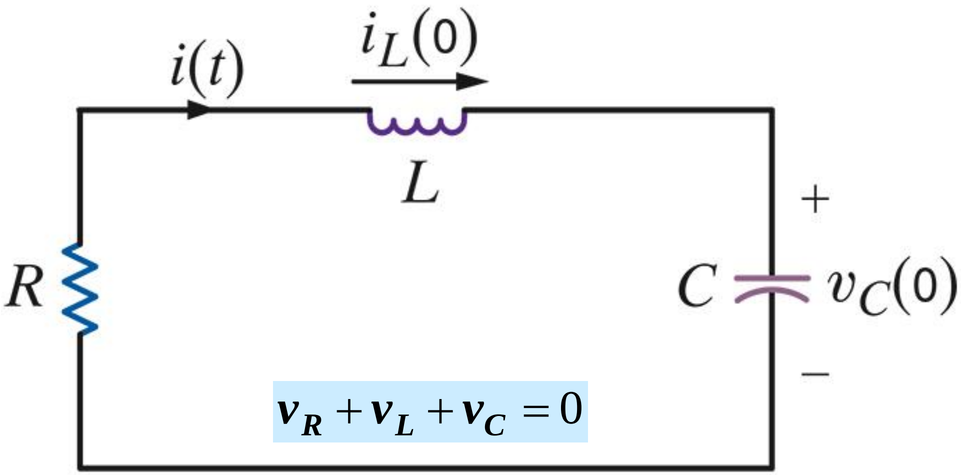
$$i(t) = e^{-3t}(A_1 \cos 4t + A_2 \sin 4t)$$

ÖRNEK -devam

i(t) AKIM CEVABINI BELİRLEYİN

$R = 6\Omega, L = 1H, C = 0.04F$

$i_L(0) = 4A;\ v_C(0) = -4V$



$v_R + v_L + v_C = 0$

ADIM 1: MODELİ ELDE EDİN√

ADIM 2: KARAKTERİSTİK DENKLEM√

ADIM 3: KÖKLERİ BULUN√

ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN√

ADIM 5: SABİTLERİ BULUN

$$i(t) = e^{-3t}(A_1 \cos 4t + A_2 \sin 4t)$$

$$i(0) = i_L(0) = 4A \Rightarrow 4 = (A_1 + 0) \Rightarrow A_1 = 4$$

$\frac{di}{dt}(0+)$'yi hesaplamak için $v_L(t) = L\frac{di}{dt}(t)$

t = 0'da anahtarlama veya süreksizlik yok.
t = 0 veya t = (0+)'yı kullanın

$$L\frac{di}{dt}(0) = -Ri(0) - v_C(0)$$

$$\frac{di}{dt}(0+) = -6 \times 4 - (-4) = -20$$

$$\frac{di}{dt}(0+) = -20$$

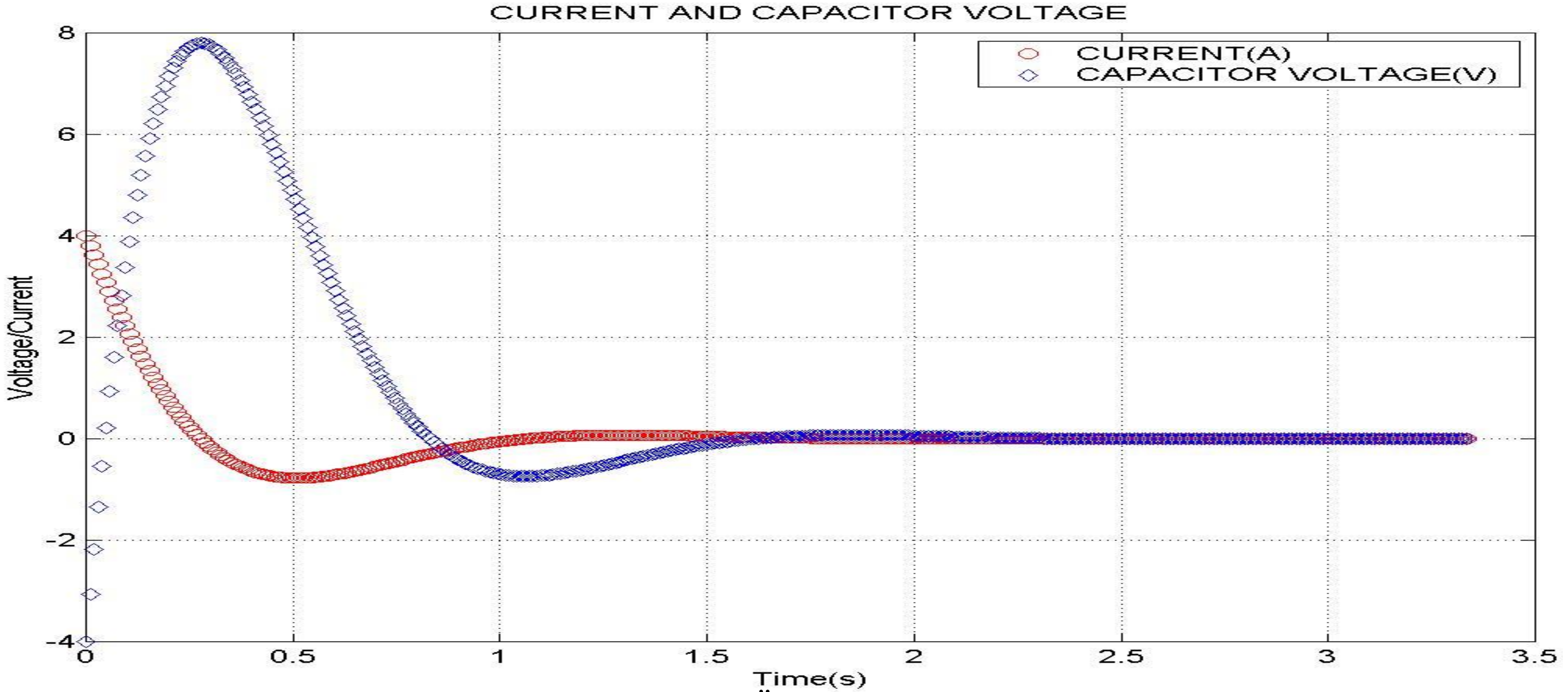
$$t = 0'da: \quad -20 = -3A_1 + 4A_2$$

$$-20 = -3 \times 4 + 4A_2 \Rightarrow A_2 = -2$$

$$i(t) = e^{-3t}(4\cos 4t - 2\sin 4t)[A];\ t > 0$$

CEVABI GÖRSELLEŞTİRMEK İÇİN MATLAB KULLANMA

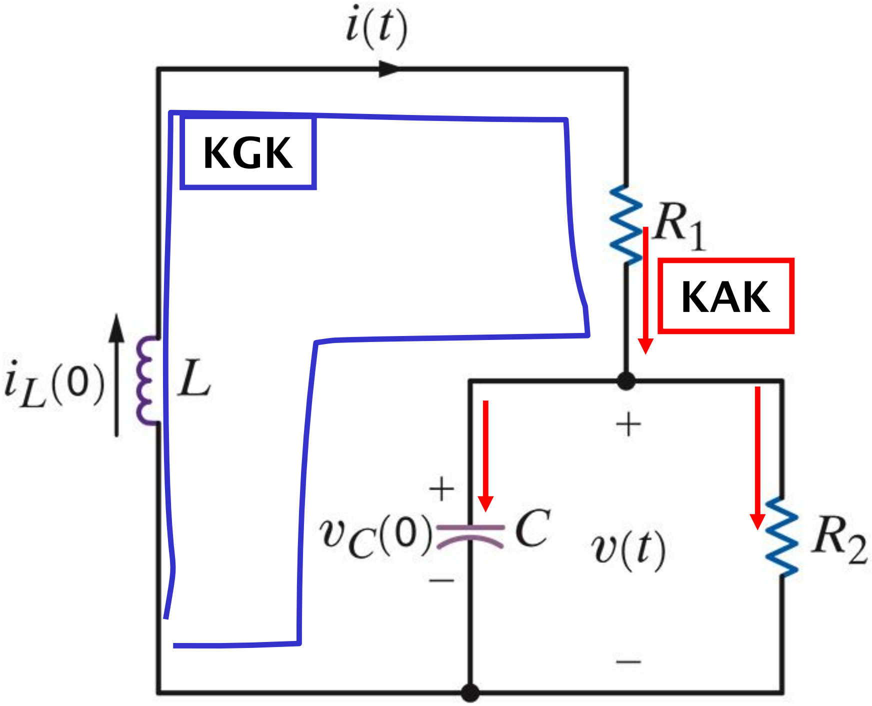
```
%script6p8.m
%displays the function i(t)=exp(-3t)(4cos(4t)-2sin(4t))
% and the function vc(t)=exp(-3t)(-4cos(4t)+22sin(4t))
% use a simle algorithm to estimate display time
tau=1/3;
tend=10*tau;
t=linspace(0,tend,350);
it=exp(-3*t).*(4*cos(4*t)-2*sin(4*t));
vc=exp(-3*t).*(-4*cos(4*t)+22*sin(4*t));
plot(t,it,'ro',t,vc,'bd'),grid,xlabel('Time(s)'),ylabel('Voltage/Current')
title('CURRENT AND CAPACITOR VOLTAGE')
legend('CURRENT(A)','CAPACITOR VOLTAGE(V)')
```



ÖRNEK *v(t)* **GERİLİM CEVABINI BELİRLEYİN**

$R_1 = 10\Omega,\ R_2 = 8\Omega,\ C = \frac{1}{8}F,\ L = 2H$

$v_C(0) = 1V,\ i_L(0) = 0.5A$



$$L\frac{di}{dt}(t) + R_1 i(t) + v(t) = 0$$

$$i(t) = \frac{v(t)}{R_2} + C\frac{dv}{dt}(t)$$

$$L\left(\frac{1}{R_2}\frac{dv}{dt}(t) + C\frac{d^2v}{dt^2}\right) + R_1\left(\frac{v(t)}{R_2} + C\frac{dv}{dt}(t)\right) + v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + \left(\frac{1}{R_2C} + \frac{R_1}{L}\right)\frac{dv}{dt}(t) + \frac{R_1 + R_2}{R_2LC}v(t) = 0$$

Değerler yerine yazıldığında DEVRE MODELİ

$$\frac{d^2v}{dt^2}(t) + 6\frac{dv}{dt}(t) + 9v(t) = 0$$

KARAKTERİSTİK DENKLEM

$$s^2 + 6s + 9 = 0$$

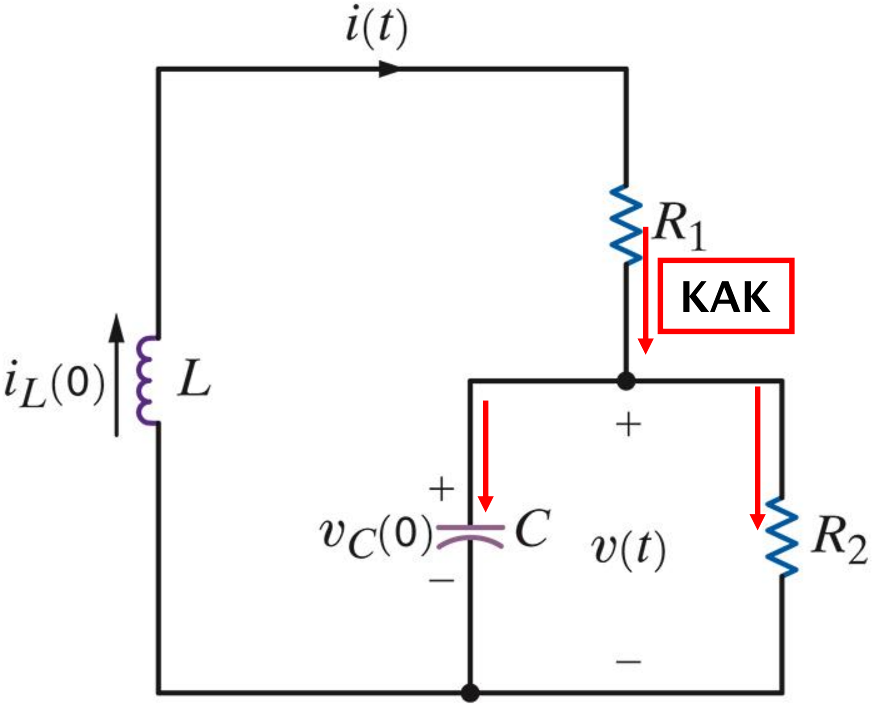
$$\omega_n = 3,\ 2\varsigma\omega_n = 6 \Rightarrow \varsigma = 1$$

ÖRNEK -devam

v(t) GERİLİM CEVABINI BELİRLEYİN

$R_1 = 10\Omega,\ R_2 = 8\Omega,\ C = \frac{1}{8}F,\ L = 2H$

$v_C(0) = 1V,\ i_L(0) = 0.5A$



KÖKLER

$$s^2 + 6s + 9 = 0 = (s+3)^2$$

$$s_1 = s_2 = -3$$

ÇÖZÜMÜN BİÇİMİ

$$v(t) = e^{-3t}(B_1 + B_2 t)$$

SABİTLER

$$v(0+) = v_c(0+) = 1V$$

$$v(0) = 1 = B_1$$

t = 0'da anahtarlama veya süreksizlik yok.
t = 0 veya t = (0+)'yı kullanın

$t = (0+)$'da KAK

$$i(0) = i_L(0) = \frac{v(0)}{R_2} + C\frac{dv}{dt}(0) \Rightarrow \frac{dv}{dt}(0) = 3$$

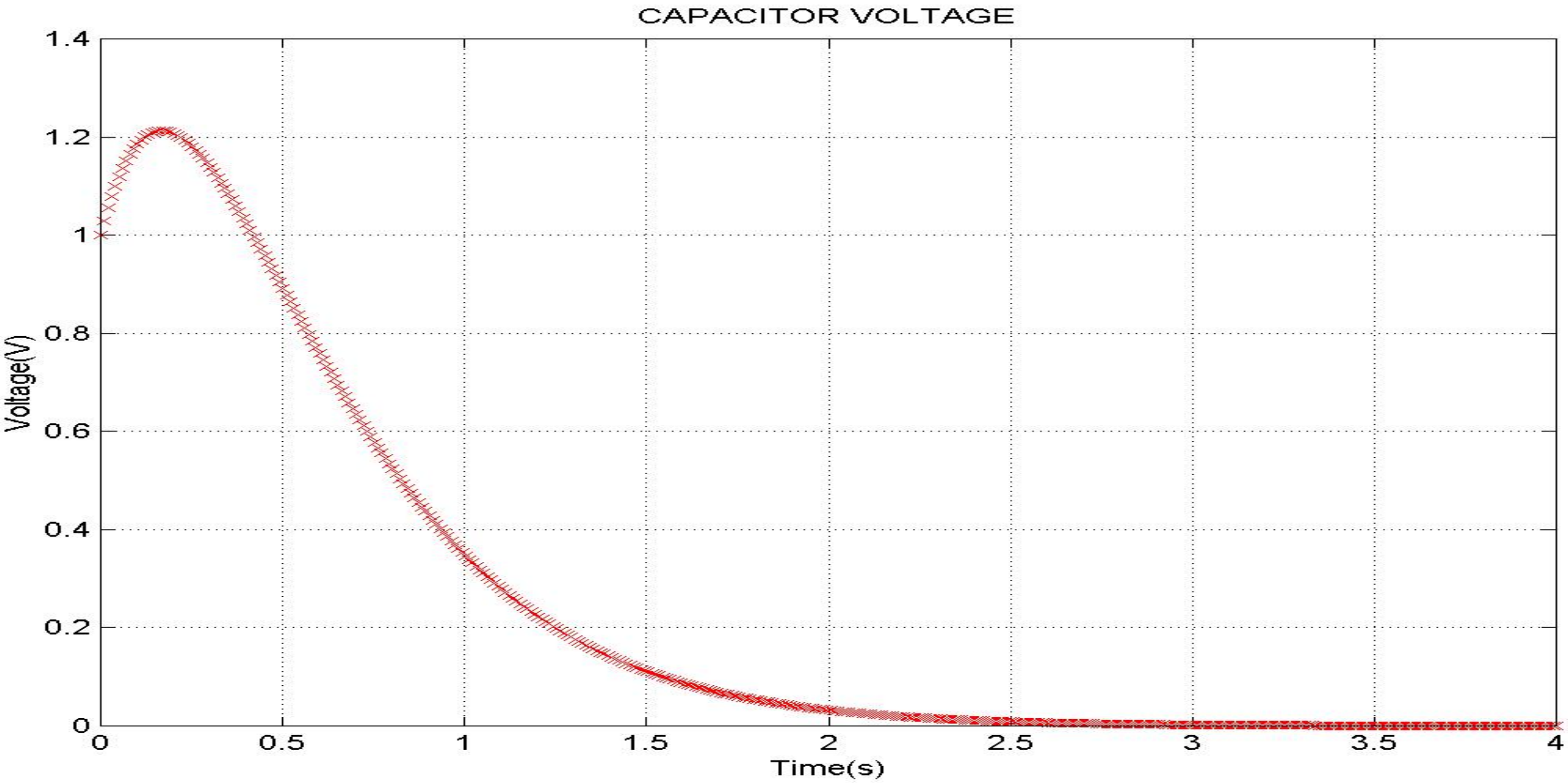
$$\frac{dv}{dt}(0) = -3e^{-3t}(B_1 + B_2 t) + B_2 e^{-3t}$$

$$\frac{dv}{dt}(0) = -3B_1 + B_2 = 3 \Rightarrow B_2 = 6$$

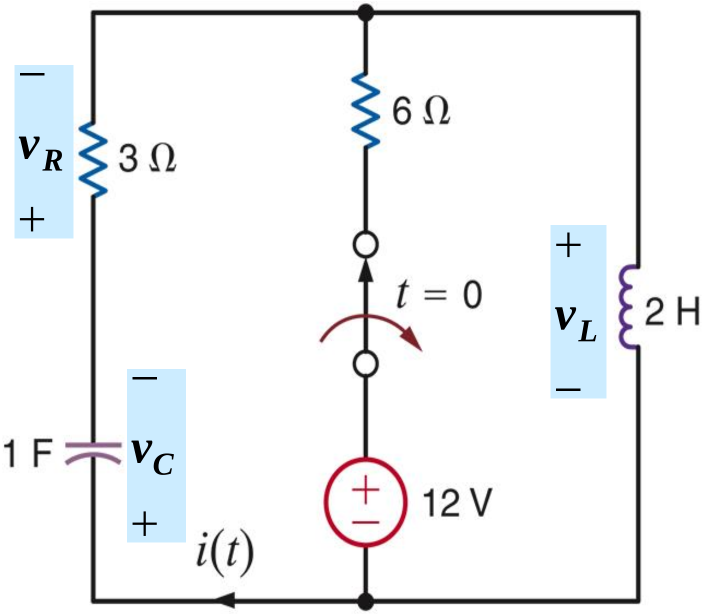
$$v(t) = e^{-3t}(1 + 6t);\ t > 0$$

CEVABI GÖRSELLEŞTİRMEK İÇİN MATLAB KULLANMA

```
%script6p9.m
%displays the function v(t)=exp(-3t)(1+6t)
tau=1/3;
tend=ceil(10*tau);
t=linspace(0,tend,400);
vt=exp(-3*t).*(1+6*t);
plot(t,vt,'rx'),grid, xlabel('Time(s)'), ylabel('Voltage(V)')
title('CAPACITOR VOLTAGE')
```



ÖRNEK *t >0 için i(t)'yi bulun*



DEVRE MODELİ

Anahtar açıldığında
devre seri RLC devresi olur

$$3i(t)+2\frac{di}{dt}(t)+v_C(0)+\int_0^t i(x)dx=0$$

$$\frac{d^2i}{dt^2}(t)+\frac{3}{2}\frac{di}{dt}(t)+\frac{1}{2}i(t)=0$$

KARAKTERİSTİK DENKLEM

$$s^2+1.5s+0.5=0$$

KÖKLER

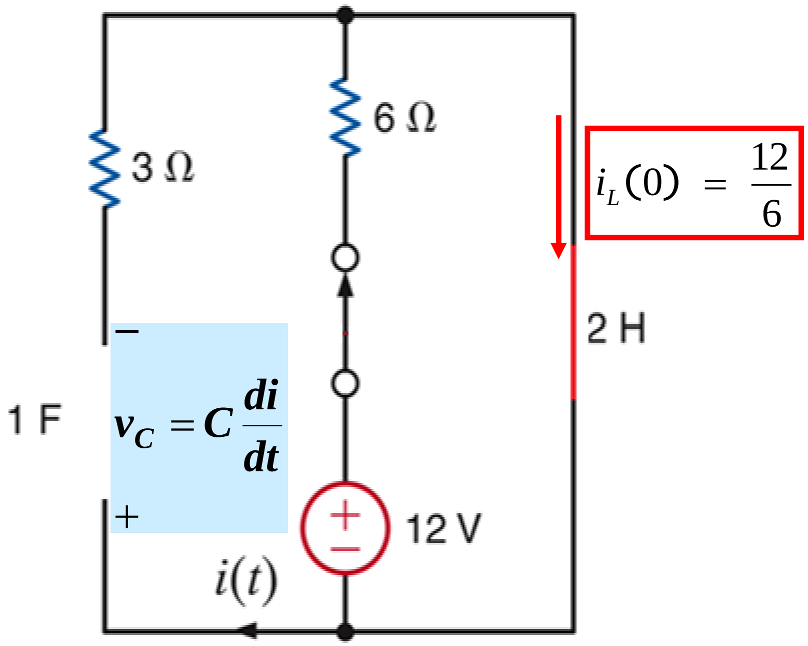
$$s_1=-1, s_2=-0.5$$

ÇÖZÜMÜN BİÇİMİ

$$i(t)=K_1e^{-t}+K_2e^{-\frac{t}{2}};\ t>0$$

ÖRNEK -devam *t >0 için i(t)'yi bulun*

Başlangıç koşullarını bulmak için t <0 'da kalıcı durum analizini kullanın



$i_L(0) = \frac{12}{6}$

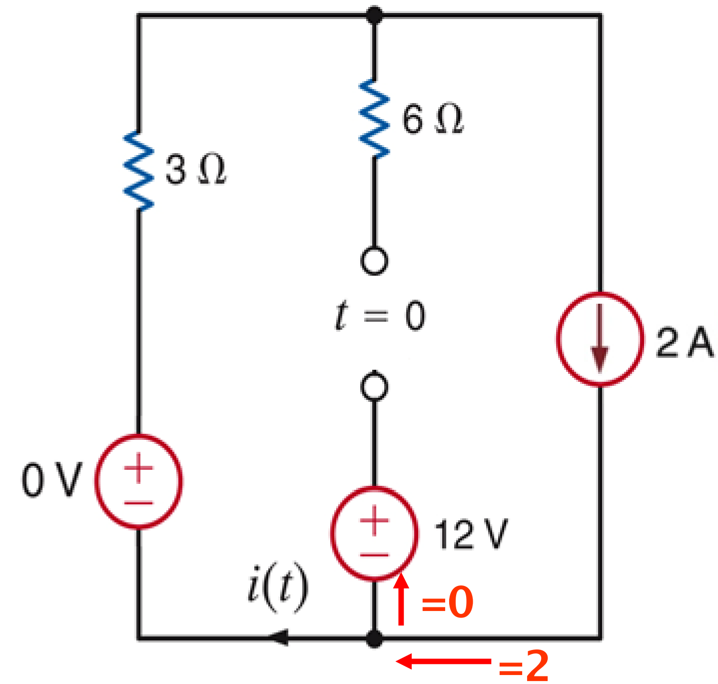
$v_C = C\frac{di}{dt}$

$v_C(0) = 0V$

$\frac{di}{dt}(0+) = 0$

$i_L(0) = 2A$

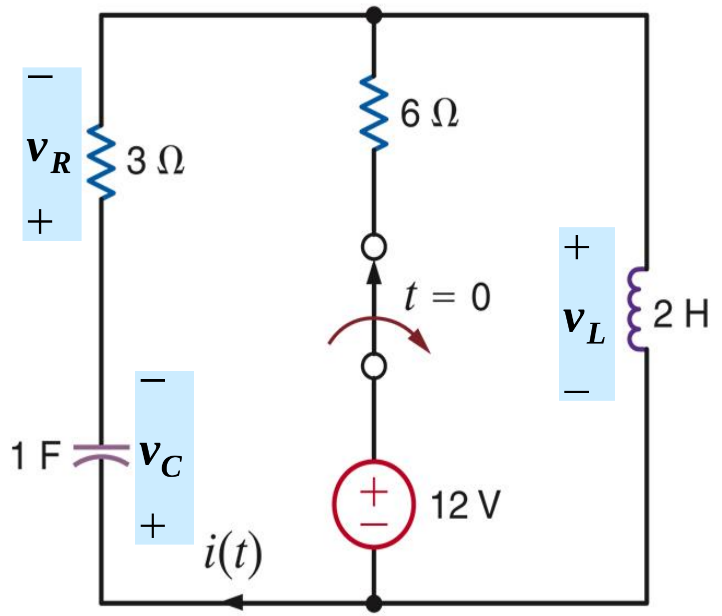
t=0+'da devreyi analiz edin



$t = (0+)$'da KAK

$i(0+) = 2A$

ÖRNEK -devam *t >0 için i(t)'yi bulun*



SABİTLER

$$v_C(0+) = 0V \qquad \frac{di}{dt}(0+) = 0 \qquad i(0+) = 2A$$

$$i(t) = K_1 e^{-t} + K_2 e^{-\frac{t}{2}}; \; t = 0+$$

$$2 = K_1 + K_2$$

$$\frac{di}{dt} = -K_1 e^{-t} - \frac{1}{2} K_2 e^{-\frac{t}{2}}; \; t = 0+$$

$$0 = -K_1 - \frac{1}{2} K_2$$

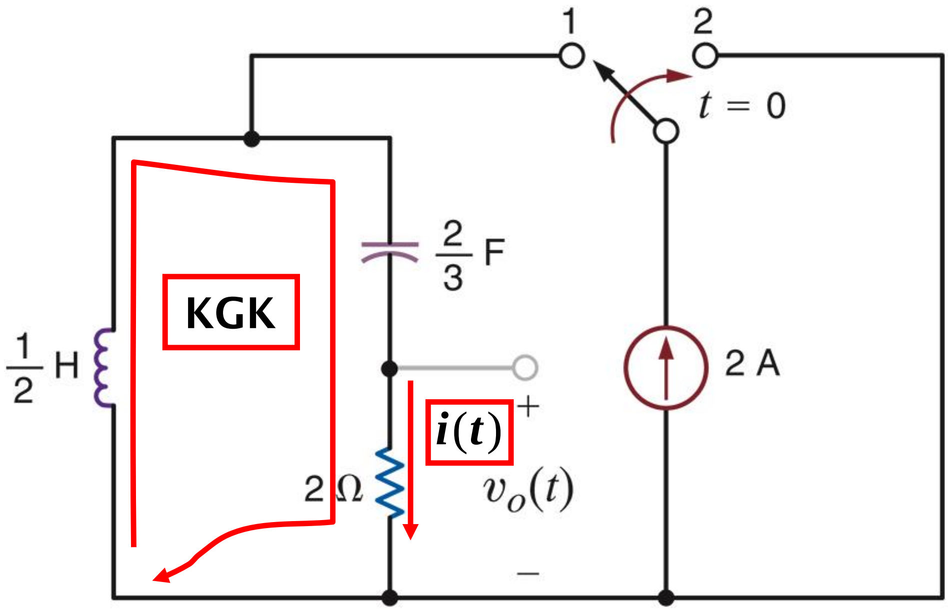
$$2 = K_1 + K_2$$

$$0 = -K_1 - \frac{1}{2} K_2$$

$$K_1 = -2, K_2 = 4$$

$$i(t) = -2e^{-t} + 4e^{-\frac{t}{2}}; \; t > 0$$

ÖRNEK *t >0 için $v_0(t)$'yi bulun*



$$\boxed{v_0(t) = 2i(t)}$$

DEVRE MODELİ

t>0 için, devre seri RLC devresi olur

$$\frac{1}{2}\frac{di}{dt}(t) + \frac{1}{2/3}\int_0^t i(x)dx + v_C(0) + 2i(t) = 0$$

$$\frac{d^2i}{dt^2}(t) + 4\frac{di}{dt}(t) + 3i(t) = 0$$

KARAKTERİSTİK DENKLEM

$$s^2 + 4s + 3 = 0$$

KÖKLER

$$s_1 = -1, s_2 = -3$$

ÇÖZÜMÜN BİÇİMİ

$$i(t) = K_1e^{-t} + K_2e^{-3t};\ t > 0$$

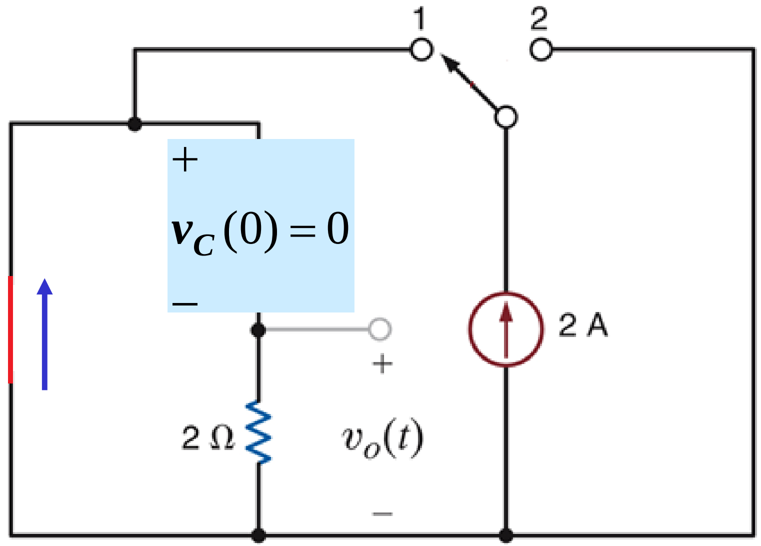
ÖRNEK -devam

t >0 için $v_0(t)$'yi bulun

$$i(t) = K_1 e^{-t} + K_2 e^{-3t};\ t > 0$$

Başlangıç koşullarını bulmak için t <0 'da kalıcı durum analizini kullanın

t=0+'da devreyi analiz edin

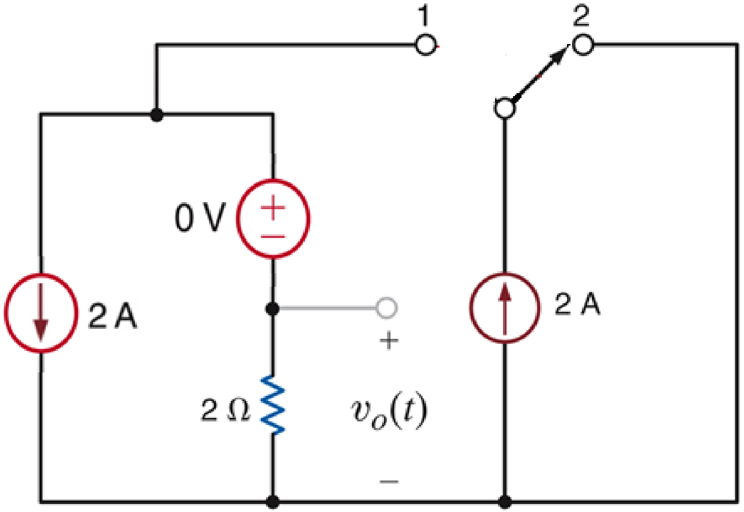


$$v_C(0) = 0$$

$$\boxed{i_L(0) = -2A}$$

$$i(0-) = -2A$$

$$v_C(0-) = 0$$



$$i(0+) = -2A \quad v_L(0+) = L\frac{di}{dt}(0+) = 0$$

$$i(0+) = -2 \Rightarrow K_1 + K_2 = -2$$

$$\frac{di}{dt}(0+) = 0 \Rightarrow -K_1 - 3K_2 = 0$$

$$K_2 = 1$$

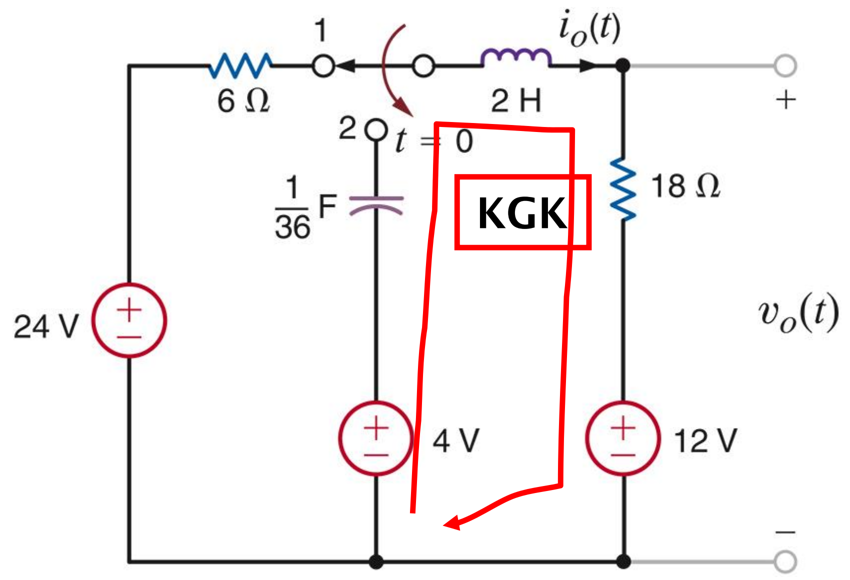
$$K_1 = -3$$

$$\therefore i(t) = -3e^{-3t} + e^{-t};\ t > 0$$

$$v_0(t) = 2\left(-3e^{-3t} + e^{-t}\right);\ t > 0$$

ÖRNEK

t >0 için $i_0(t)$ ve $v_0(t)$'yi bulun



DEVRE MODELİ

$$-4+\frac{1}{1/36}\int_0^t i(x)dx+v_C(0)+2\frac{di}{dt}(t)+18i(t)+12=0$$

$$\frac{d^2i}{dt^2}(t)+9\frac{di}{dt}(t)+18i(t)=0$$

$$v_0(t)=18i_0(t)+12(V)$$

KARAKTERİSTİK DENKLEM

$$s^2+9s+18=0$$

KÖKLER

$$s_1=-3, s_2=-6$$

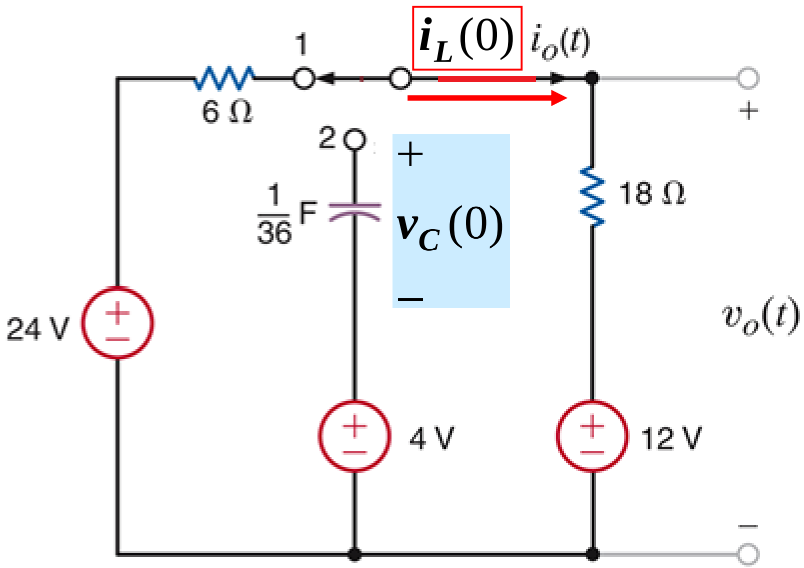
ÇÖZÜMÜN BİÇİMİ

$$i_0(t)=K_1e^{-3t}+K_2e^{-6t};\ t>0$$

ÖRNEK -devam

t >0 için $i_0(t)$ ve $v_0(t)$'yi bulun

Başlangıç şartlarını bulmak için t <0 'da kalıcı durum analizini kullanın



$$v_C(0-) = 0$$

$$i_L(0-) = 0.5A$$

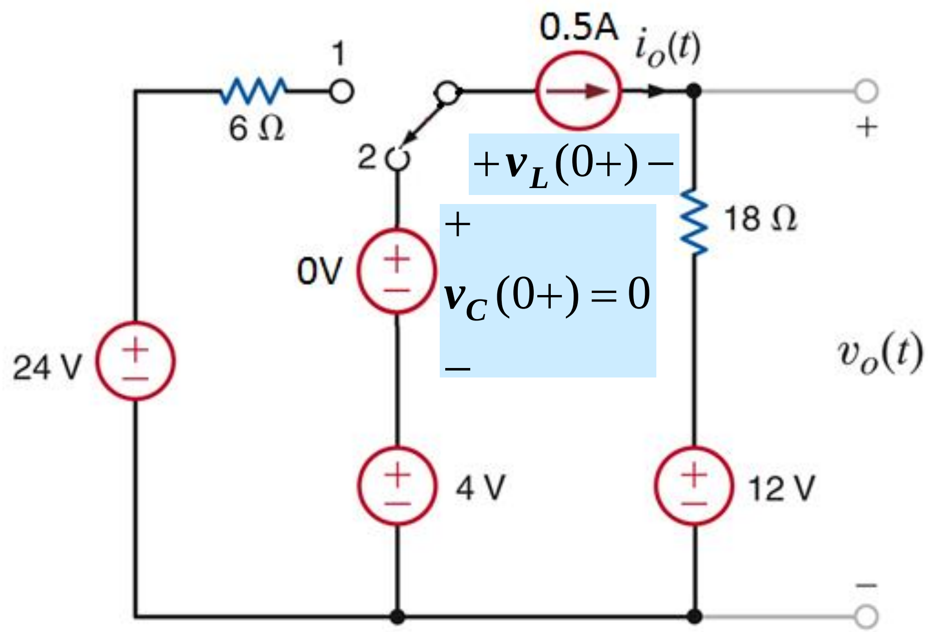
ÖRNEK -devam

t >0 için $i_0(t)$ ve $v_0(t)$'yi bulun

$$v_0(t) = 18i_0(t) + 12(V)$$

$$i_0(t) = K_1 e^{-3t} + K_2 e^{-6t};\ t > 0$$

t=0+'da devreyi analiz edin



$v_C(0-) = 0$ $i_L(0-) = 0.5A$

$$i_0(0+) = i_L(0+) = 0.5(A)$$

$$i_0(0+) = 0.5 = K_1 + K_2$$

$$v_L(0+) = L\frac{di_L}{dt}(0+) = L\frac{di_0}{dt}(0+)$$

$$-4 + 2\frac{di_L}{dt}(0+) + 18i_L(0+) + 12 = 0$$

$$\frac{di_0}{dt}(0+) = -17/2$$

$$\frac{di_0}{dt} = -3K_1 e^{-3t} - 6K_2 e^{-6t};\ t = 0+$$

$$\frac{di_0}{dt}(0+) = -17/2 = -3K_1 - 6K_2$$

$$-17/2 = -3K_1 - 6K_2$$
$$0.5 = K_1 + K_2$$

$$K_1 = -\frac{11}{6}; \quad K_2 = \frac{14}{6}$$

$$i_0(t) = -\frac{11}{6}e^{-3t} + \frac{14}{6}e^{-6t};\ t > 0$$

$$v_0(t) = 12 + 18i_0(t);\ t > 0$$