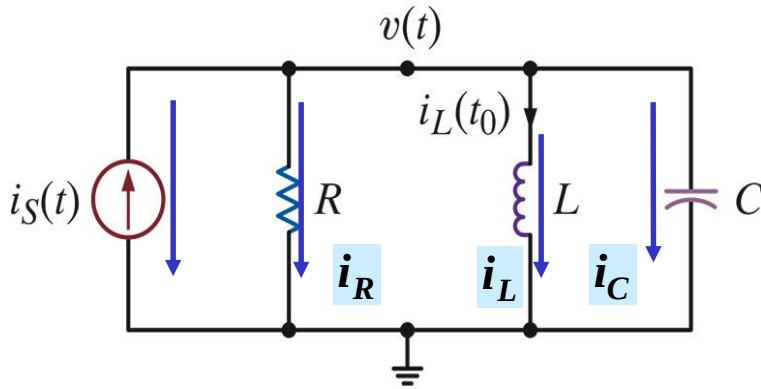


İKİNCİ DERECE DENKLEMLER

TEMEL DEVRE DENKLEMİ



Tek Düğüm Çiftli Devre: KAK kullan

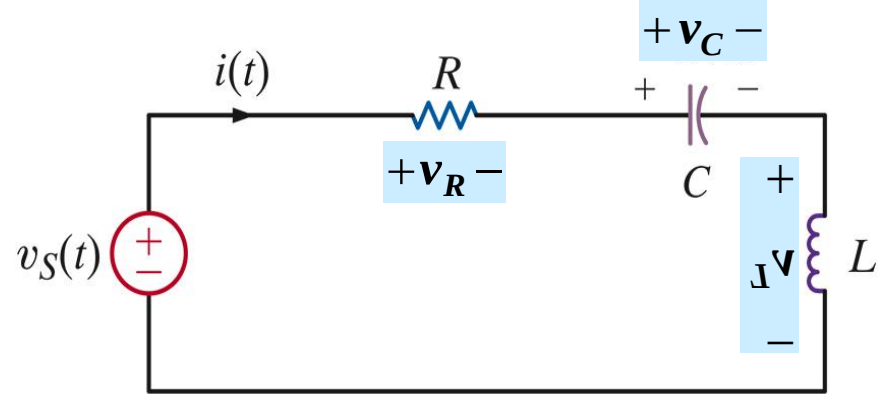
$$-i_S + i_R + i_L + i_C = 0$$

$$i_R = \frac{v(t)}{R}; \quad i_L = \frac{1}{L} \int_{t_0}^t v(x) dx + i_L(t_0); \quad i_C = C \frac{dv}{dt}(t)$$

$$\frac{v}{R} + \frac{1}{L} \int_{t_0}^t v(x) dx + i_L(t_0) + C \frac{dv}{dt}(t) = i_S$$

Türev alındığında

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} = \frac{di_S}{dt}$$



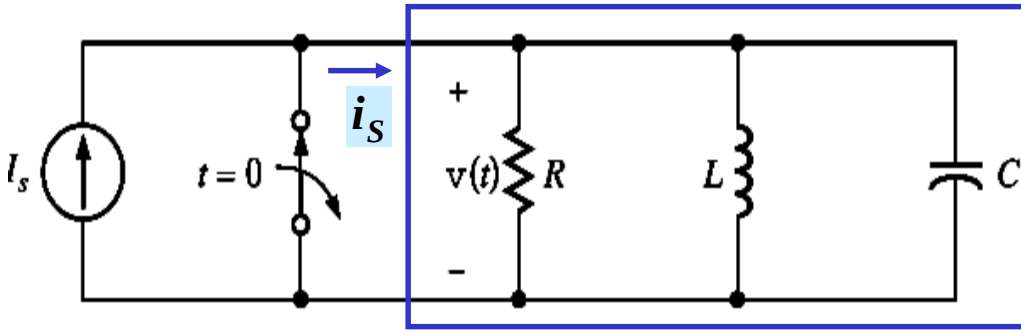
Tek Gözlü Devre: KKG kullan

$$-v_S + v_R + v_C + v_L = 0$$

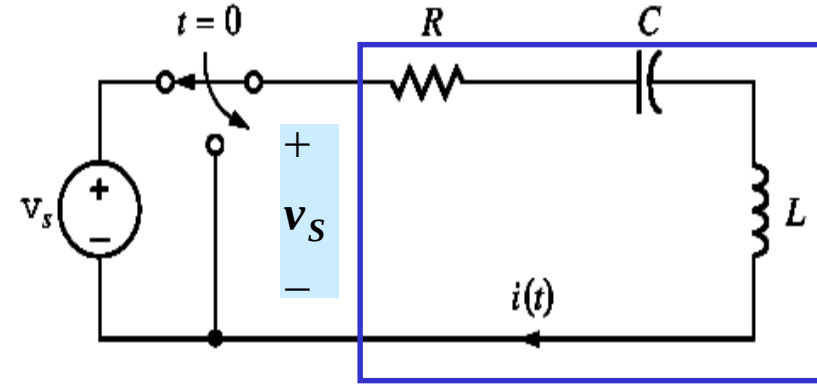
$$v_R = Ri; \quad v_C = \frac{1}{C} \int_{t_0}^t i(x) dx + v_C(t_0); \quad v_L = L \frac{di}{dt}(t)$$

$$Ri + \frac{1}{C} \int_{t_0}^t i(x) dx + v_C(t_0) + L \frac{di}{dt}(t) = v_S$$

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \frac{dv_S}{dt}$$

ÖRNEK **$v(t)$ ve $i(t)$ için Diferansiyel Denklemleri Yazın**

$$i_s(t) = \begin{cases} 0 & t < 0 \\ I_S & t > 0 \end{cases} \quad \frac{di_s}{dt}(t) = 0; t > 0$$



$$v_s(t) = \begin{cases} V_S & t < 0 \\ 0 & t > 0 \end{cases} \quad \frac{dv_s}{dt}(t) = 0; t > 0$$

RLC PARALEL DEVRE İÇİN MODEL

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} = \frac{di_s}{dt}$$

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} = 0$$

RLC SERİ DEVRE İÇİN MODEL

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \frac{dv_s}{dt}$$

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0$$

CEVAP DENKLEMİ

AŞAĞIDAKİ DENKLEMİN ÇÖZÜMÜ

$$\frac{d^2 x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_0 x(t) = f(t)$$

Biliyoruz ki: $x(t) = x_p(t) + x_c(t)$

x_p özel çözüm (zorlanmış)

x_c tamamlayıcı çözüm (doğal)

Doğal (Tamamlayıcı) çözüm:

$$\frac{d^2 x_c(t)}{dt^2} + a_1 \frac{dx_c(t)}{dt} + a_0 x_c(t) = 0$$

Zorlanmış çözüm:

Eğer zorlama fonksiyonu bir sabitse; $f(t) = A$

$$\frac{d^2 x_p(t)}{dt^2} + a_1 \frac{dx_p(t)}{dt} + a_0 x_p(t) = A$$

$$\frac{d^2 x_p}{dt^2} = 0, \quad \frac{dx_p}{dt} = 0, \Rightarrow a_0 x_p = A \Rightarrow x_p = \frac{A}{a_0}$$

Herhangi bir zorlama fonksiyonu $f(t) = A$ ise;

$$\text{Tam çözüm : } x(t) = \frac{A}{a_0} + x_c(t)$$

DOĞAL (HOMOJEN) DENKLEM

$$\frac{d^2 x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_0 x(t) = 0$$

NORMALIZE EDİLMİŞ FORMU

$$\frac{d^2 x(t)}{dt^2} + 2\zeta\omega_n \frac{dx(t)}{dt} + \omega_n^2 x(t) = 0$$

ω_n (sönümsüz) doğal frekans
 ζ sönüm oranı

KARAKTERİSTİK DENKLEM

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$a_0 = \omega_n^2 \Rightarrow \omega_n = \sqrt{a_0}$$

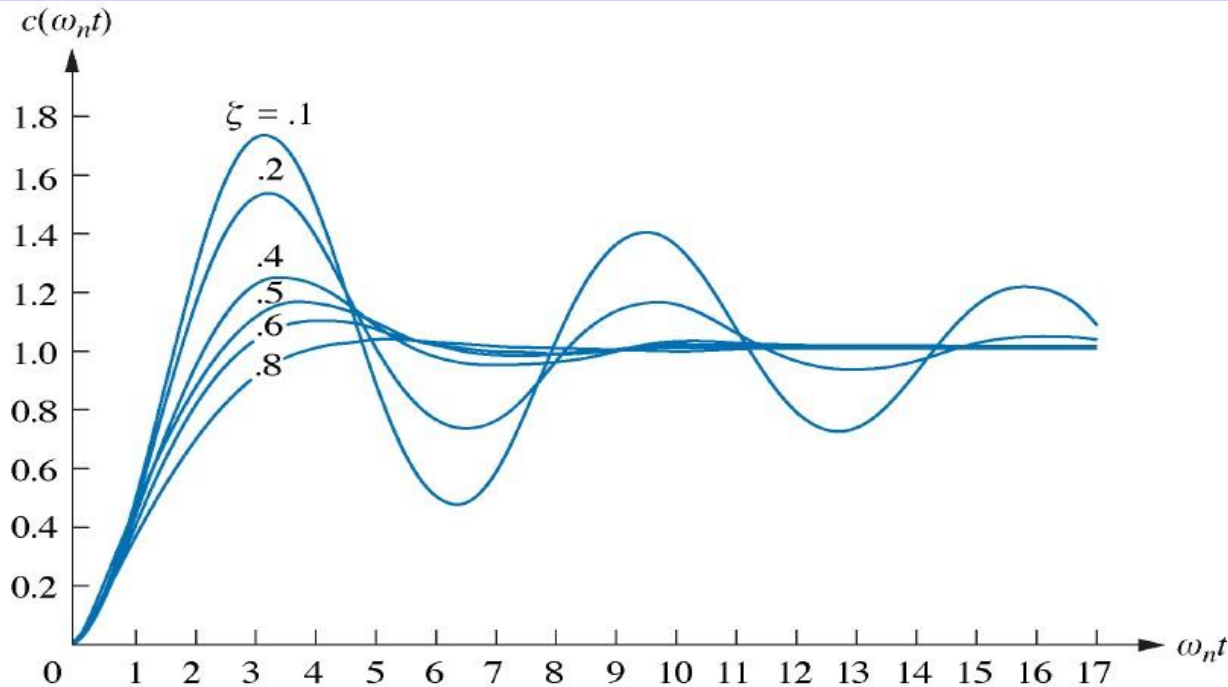
$$a_1 = 2\zeta\omega_n \Rightarrow \zeta = \frac{a_1}{2\sqrt{a_0}}$$

İkinci Dereceden Sistemler

İkinci dereceden sistemlerin cevabını iki parametre kullanarak karakterize edebiliriz: ω_n ve ζ

Doğal Frekans, ω_n : Sönümsüz osilasyon frekansıdır. Örneğin, direnci kısa devre yapılmış bir RLC devresinin veya sönümleyicisiz mekanik bir sistemin doğal frekansı. Sönümsüz bir sistem doğal frekans ile tanımlanmaktadır.

Sönümlleme Oranı, ζ : Sönümlleme miktarını ölçer. Az sönümlü sistemler için ζ sönümlleme oranı $[0, 1]$ aralığında bulunur:



RLC SERİ DEVRE İÇİN MODEL

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \frac{dv_s}{dt}$$

1- Karakteristik denklemi elde etmek için giriş sıfırlanır. (homojen denklem)

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0$$

2- En büyük türev sabitini bir yapmak için denklem tekrar düzenlenir.

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

3- Standart cevap ile karşılaştırılır.

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = 0 \quad \Rightarrow \quad \frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\omega_n = \sqrt{\frac{1}{LC}}$$

$$2\zeta\omega_n = \frac{R}{L}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0}} = \frac{\frac{R}{L}}{2\sqrt{\frac{1}{LC}}}$$

ÖRNEK

KARAKTERİSTİK DENKLEMİ, SÖNÜM ORANINI VE DOĞAL FREKANSI BELİRLEYİNİZ

$$4 \frac{d^2 x(t)}{dt^2} + 8 \frac{dx(t)}{dt} + 16x(t) = 0$$

İKİNCİ DERECEDEKİ TÜREVİN KATSAYISI BİR OLMALIDIR.

$$\frac{d^2 x}{dt^2}(t) + 2 \frac{dx}{dt}(t) + 4x(t) = 0$$

KARAKTERİSTİK DENKLEM

$$s^2 + 2s + 4 = 0$$

SÖNÜM ORANI, DOĞAL FREKANS

$$\frac{d^2 x}{dt^2}(t) + 2 \frac{dx}{dt}(t) + 4x(t) = 0$$

$$2\zeta\omega_n \quad \omega_n^2 \Rightarrow \omega_n = 2$$

⇓

$$\zeta = 0.5$$

DOĞAL (HOMOJEN) DENKLEMİN ANALİZİ

NORMALIZE EDİLMİŞ FORMU

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_n \frac{dx(t)}{dt} + \omega_n^2 x(t) = 0$$

Eğer

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \text{ ise,}$$

çözüm $x(t) = Ke^{st}$ dir

Eğer s , *karakteristik denklemin* çözümü ise

$$\text{ISPAT: } \frac{dx(t)}{dt} = sKe^{st}; \quad \frac{d^2x}{dt^2} = s^2Ke^{st}$$

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_n \frac{dx(t)}{dt} + \omega_n^2 x(t) = (s^2 + 2\zeta\omega_n s + \omega_n^2)Ke^{st}$$

KARAKTERİSTİK DENKLEM

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

(sistemin modeli)

DURUM 1: $\zeta > 1$ (gerçek ve ayrı kökler) **Aşırı Sönümlü**

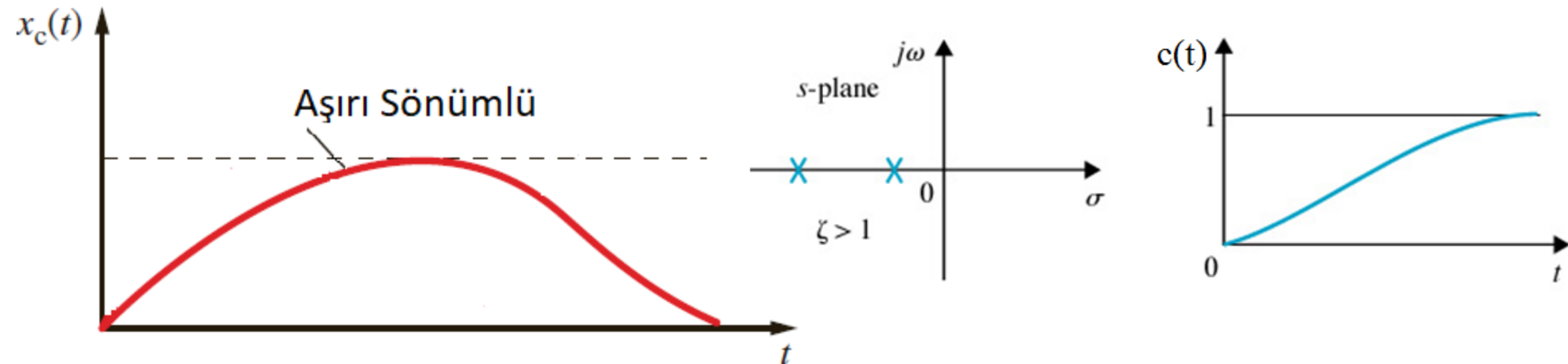
$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$s_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

$$s_1 = -\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1}$$

$$x(t) = K_1 e^{-(\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1})t} + K_2 e^{-(\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1})t}$$



DURUM 2: $\zeta < 1$ (karmasik kökler) **Az Sönümlü**

$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$$

$$s_{1,2} = -\sigma \pm j \omega_d$$

$$s_1 = -\zeta \omega_n + j \omega_n \sqrt{1 - \zeta^2} = -\sigma + j \omega_d$$

$$s_2 = -\zeta \omega_n - j \omega_n \sqrt{1 - \zeta^2} = -\sigma - j \omega_d$$

$$x(t) \text{ gerçek} \Rightarrow K_2 = K_1^*$$

ω_d = sönümlü osilasyon frekansı

σ = sönüm katsayısı

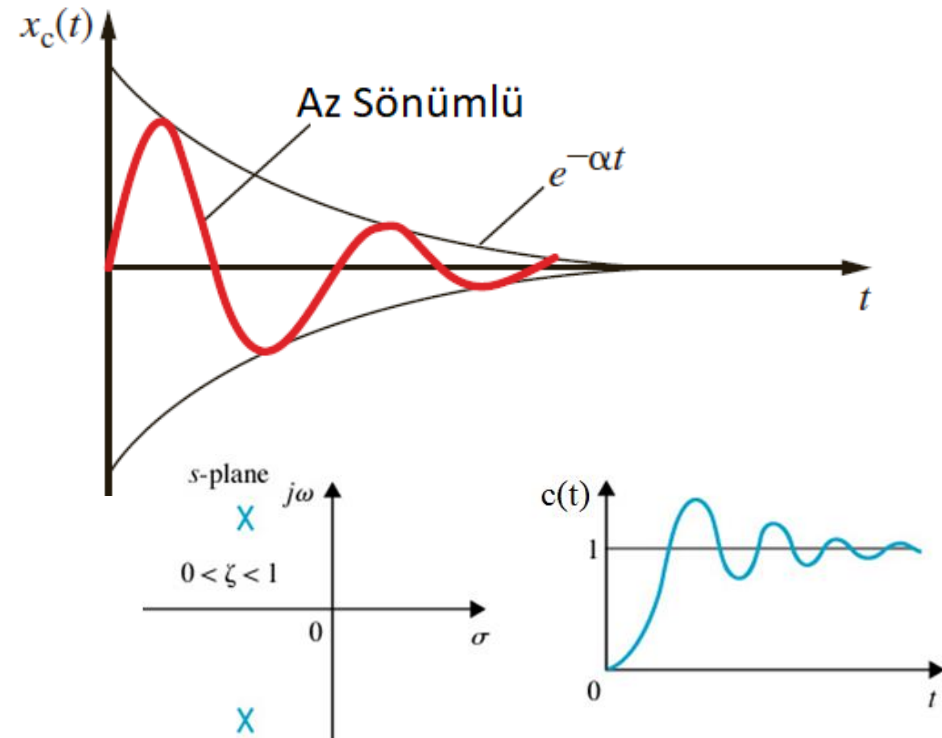
$$x(t) = e^{-\sigma t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

$$\text{IP UCU: } e^{st} = e^{-(\zeta \omega_n \pm j \omega_d)t} = e^{-\zeta \omega_n t} e^{\mp j \omega_d t}$$

$$e^{\mp j \omega_d t} = \cos \omega_d t \mp j \sin \omega_d t$$

$$\text{VARSAYIN } K_1 = (A_1 + j A_2) / 2$$

$$\left. \begin{array}{l} K_2 = K_1^* \\ s = -\sigma \pm j \omega_d \end{array} \right\} \Rightarrow x(t) = 2 \operatorname{Re} [K_1 e^{-(\sigma + j \omega_d)t}]$$



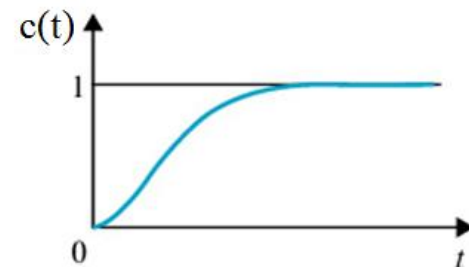
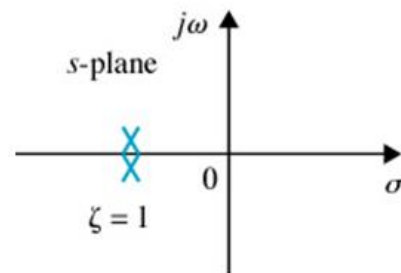
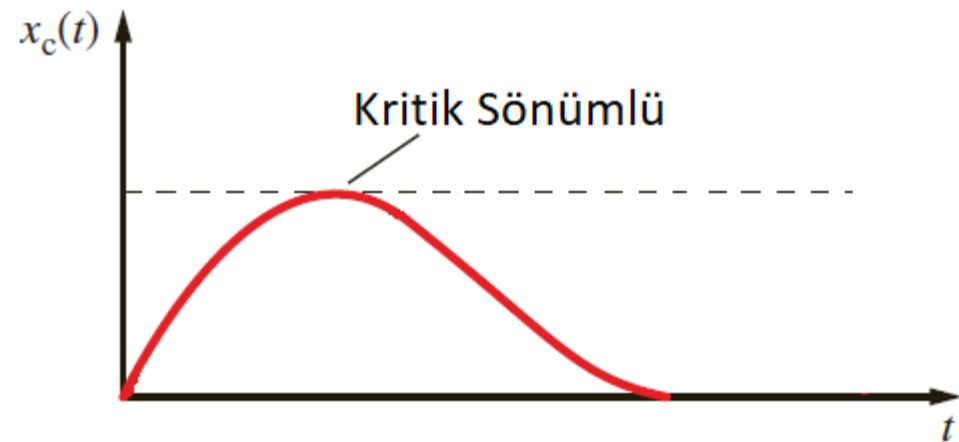
DURUM 3: $\zeta=1$ (gerçek ve katlı kökler) **Kritik Sönümlü**

$$x(t) = e^{-\zeta\omega_n t} (B_1 + B_2 t)$$

$$s = -\zeta\omega_n \quad s_1 = s_2 = -\zeta\omega_n$$

IP UCU: Eger

$(s^2 + 2\zeta\omega_n s + \omega_n^2 = 0)$ VE $(2s + 2\zeta\omega_n = 0)$ ise,
 te^{st} çözümdür



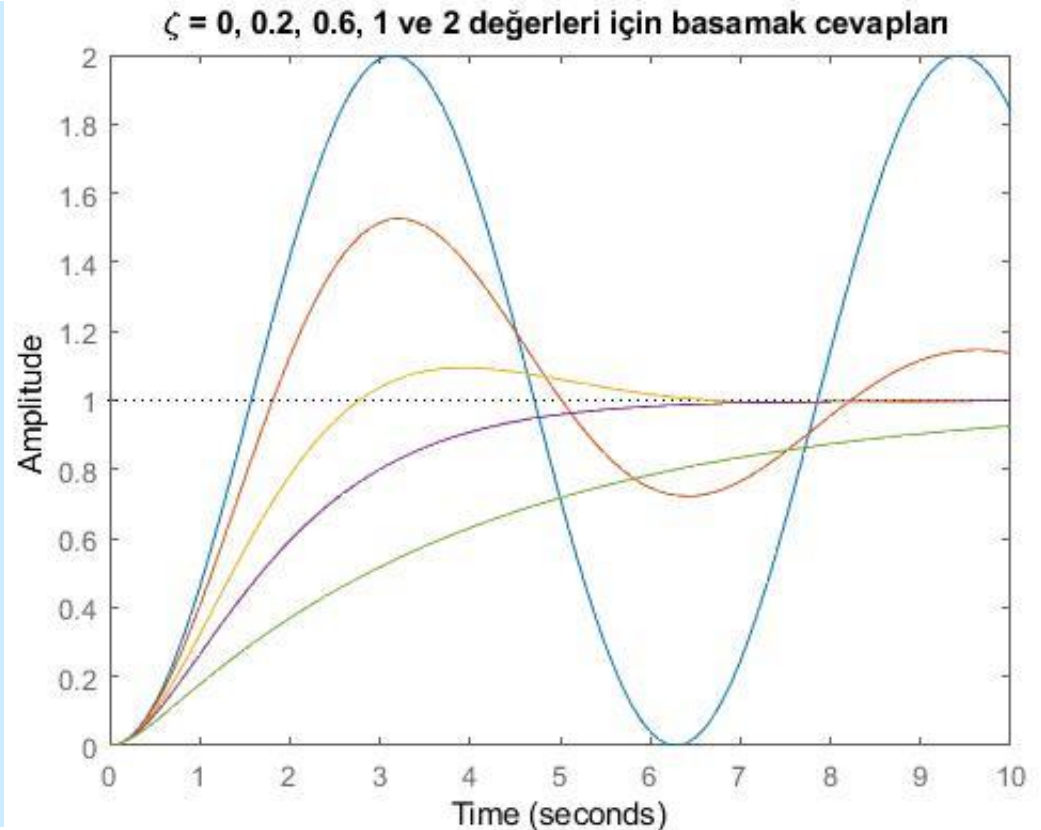
İkinci Dereceden Sistemler

Farklı Sönüm oranları için basamak cevapları

```
t = [0:0.1:10];  
pay = [1];  
zeta1 = 0.0; payda1 = [1 2*zeta1 1];  
zeta2 = 0.2; payda2 = [1 2*zeta2 1];  
zeta3 = 0.6; payda3 = [1 2*zeta3 1];  
zeta4 = 1.0; payda4 = [1 2*zeta4 1];  
zeta5 = 2.0; payda5 = [1 2*zeta5 1];
```

```
step(pay,payda1,t); hold on;  
step(pay,payda2,t); hold on;  
step(pay,payda3,t); hold on;  
step(pay,payda4,t); hold on;  
step(pay,payda5,t)
```

```
title('\zeta = 0, 0.2, 0.6, 1 ve 2  
değerleri için basamak cevapları')
```



PROBLEM-SOLVING STRATEGY

- STEP 1.** Write the differential equation that describes the circuit.
- STEP 2.** Derive the characteristic equation, which can be written in the form $s^2 + 2\zeta\omega_0s + \omega_0^2 = 0$, where ζ is the damping ratio and ω_0 is the undamped natural frequency.
- STEP 3.** The two roots of the characteristic equation will determine the type of response. If the roots are real and unequal (i.e., $\zeta > 1$), the network response is overdamped. If the roots are real and equal (i.e., $\zeta = 1$), the network response is critically damped. If the roots are complex (i.e., $\zeta < 1$), the network response is underdamped.
- STEP 4.** The damping condition and corresponding response for the aforementioned three cases outlined are as follows:
- Overdamped: $x(t) = K_1e^{-(\zeta\omega_0 - \omega_0\sqrt{\zeta^2-1})t} + K_2e^{-(\zeta\omega_0 + \omega_0\sqrt{\zeta^2-1})t}$
- Critically damped: $x(t) = B_1e^{-\zeta\omega_0t} + B_2te^{-\zeta\omega_0t}$
- Underdamped: $x(t) = e^{-\sigma t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$, where $\sigma = \zeta\omega_0$, and $\omega_d = \omega_0\sqrt{1 - \zeta^2}$
- STEP 5.** Two initial conditions, either given or derived, are required to obtain the two unknown coefficients in the response equation.

ÖRNEK**ÇÖZÜMÜN GENEL BİÇİMİNİ BELİRLEYİN**

$$\frac{d^2 x}{dt^2}(t) + 4 \frac{dx}{dt}(t) + 4x(t) = 0$$

KARAKTERİSTİK DENKLEM

$$s^2 + 4s + 4 = 0$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2$$

$$2\zeta\omega_n = 4 \Rightarrow \zeta = 1$$

$$s^2 + 4s + 4 = 0 \Rightarrow (s + 2)^2 = 0$$

Kökler gerçekte ve eşit**Bu kritik sönümlü bir sistemdir (DURUM 3)**

$$x(t) = e^{st} (B_1 + B_2 t)$$

$$x(t) = e^{-2t} (B_1 + B_2 t)$$

ÖRNEK ÇÖZÜMÜN GENEL BİÇİMİNİ BELİRLEYİN

$$4\frac{d^2x}{dt^2}(t) + 8\frac{dx}{dt}(t) + 16x(t) = 0$$

ikinci dereceden türev katsayısı ile bölün

$$\frac{d^2x}{dt^2}(t) + 2\frac{dx}{dt}(t) + 4x(t) = 0$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2$$

$$2\zeta\omega_n = 2 \Rightarrow \zeta = 0.5$$

KARAKTERİSTİK DENKLEM

$$s^2 + 2s + 4 = 0$$

$$\rightarrow s^2 + 2s + 4 = (s+1)^2 + 3 = 0 \Rightarrow s = -1 \pm j\sqrt{3}$$

Kökler karmaşık eşleniktir

ω_d

Bu az sönümlü bir sistemdir (DURUM 2)

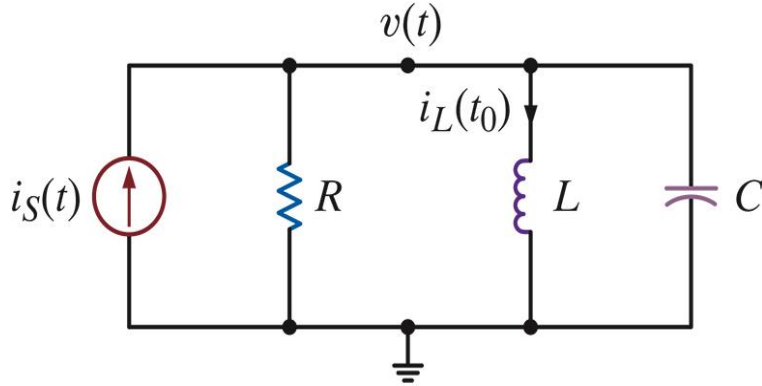
$$\sigma = \zeta\omega_n = 1; \quad \omega_d = \omega_n\sqrt{1-\zeta^2} = 2\sqrt{1-0.25} = \sqrt{3}$$

$$x(t) = e^{-\sigma t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

$$x(t) = e^{-t} (A_1 \cos \sqrt{3}t + A_2 \sin \sqrt{3}t)$$

ÖRNEK**Çözümün biçimini belirleyin**PARALEL *RLC* DEVRESİ

$$R = 1\Omega, L = 2H, C = 2F$$

**HOMOJEN DENKLEM**

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} = 0$$

$$2 \frac{d^2 v}{dt^2} + \frac{dv}{dt} + \frac{v}{2} = 0$$

$$\frac{d^2 v}{dt^2} + \frac{1}{2} \frac{dv}{dt} + \frac{v}{4} = 0$$

$$s^2 + \frac{1}{2}s + \frac{1}{4} = (s + \frac{1}{4})^2 + \frac{3}{16} = 0$$

$$\omega_n = \frac{1}{2}; \zeta \omega_n = \frac{1}{4} \Rightarrow \zeta = \frac{1}{2}$$

$$\sigma = -\frac{1}{4} \quad \omega_d = \omega_n \sqrt{1 - \zeta^2} = \frac{1}{2} \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{4}$$

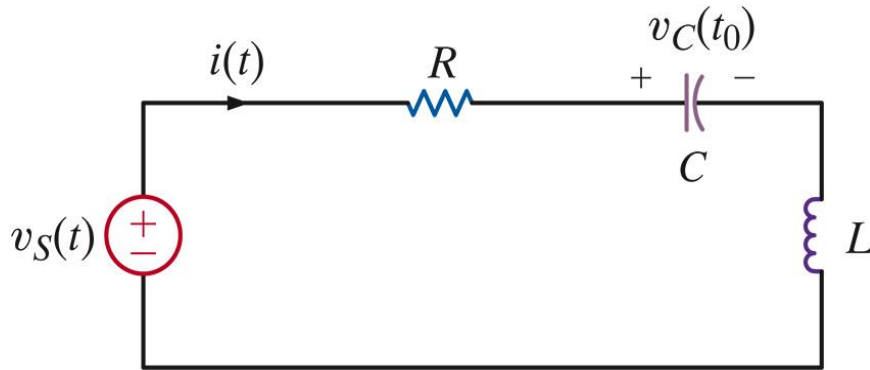
$$v_c(t) = e^{-\frac{t}{4}} \left(A_1 \cos \frac{\sqrt{3}}{4} t + A_2 \sin \frac{\sqrt{3}}{4} t \right)$$

ÖRNEK

Verilen kapasitans değerleri için sistem cevaplarını sınıflandırın

SERİ RLC DEVRESİ

$$R = 2\Omega; L = 1H; C = 0.5F, C = 1F, C = 2F$$

**HOMOJEN DENKLEM**

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0 : / L \quad \& \text{değerler yerine yazıldığında}$$

$$\frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + \frac{i}{C} = 0$$

$$s^2 + 2s + \frac{1}{C} = 0$$

$$\omega_n = \frac{1}{\sqrt{C}}; 2\zeta\omega_n = 2 \Rightarrow \zeta = \sqrt{C}$$

C=0.5 az sönümlü
C=1.0 kritik sönümlü
C=2.0 aşırı sönümlü

$$\text{diskriminant} = 4 - \frac{4}{C}$$

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2x}{dt^2}(t) + 2\zeta\omega_n \frac{dx}{dt}(t) + \omega_n^2 x(t) = A$$

Biliyoruz ki: $x(t) = x_p(t) + x_c(t)$ x_p özel çözüm (zorlanmış) x_c tamamlayıcı çözüm (doğal)

$$x_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x_p(t) = \frac{A}{\omega_n^2}$$

$$x(t) = \frac{A}{\omega_n^2} + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x(0+) - \frac{A}{\omega_n^2} = K_1 + K_2$$

$$\frac{dx}{dt}(0+) = s_1 K_1 + s_2 K_2$$

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2 x}{dt^2}(t) + 2\zeta\omega_n \frac{dx}{dt}(t) + \omega_n^2 x(t) = A$$

Biliyoruz ki: $x(t) = x_p(t) + x_c(t)$ x_p özel çözüm (zorlanmış) x_c tamamlayıcı çözüm (doğal)

$$x_c(t) = e^{-\zeta\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

$$x_p(t) = \frac{A}{\omega_n^2}$$

$$x(t) = \frac{A}{\omega_n^2} + e^{-\zeta\omega_n t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

$$x(0+) - \frac{A}{\omega_n^2} = A_1$$

$$\frac{dx}{dt}(0+) = -\zeta\omega_n A_1 + \omega_d A_2$$

SABİTLERİN BELİRLENMESİ

Denklemin Normalize Edilmiş Biçimi

$$\frac{d^2x}{dt^2}(t) + 2\zeta\omega_n \frac{dx}{dt}(t) + \omega_n^2 x(t) = A$$

$$x_c(t) = e^{-\zeta\omega_n t} (B_1 + B_2 t)$$

$$x_p(t) = \frac{A}{\omega_n^2}$$

$$x(t) = \frac{A}{\omega_n^2} + e^{-\zeta\omega_n t} (B_1 + B_2 t)$$

$$x(0+) - \frac{A}{\omega_n^2} = B_1$$

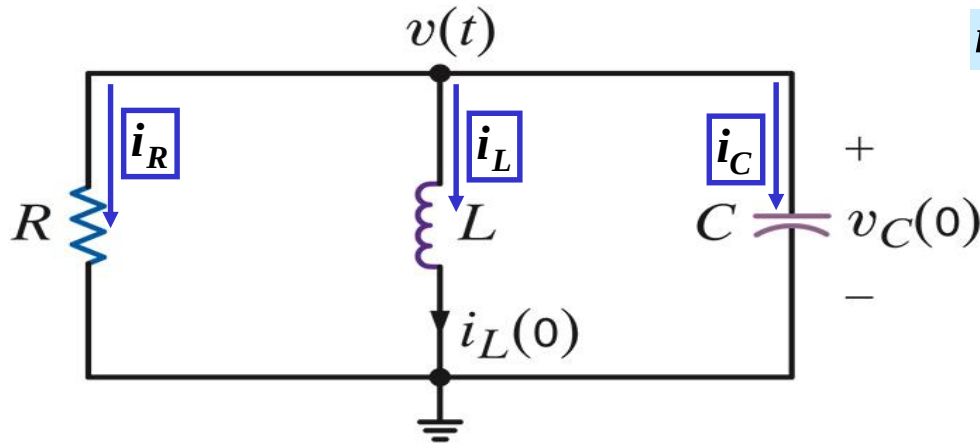
$$\frac{dx}{dt}(0+) = -\zeta\omega_n B_1 + B_2$$

Biliyoruz ki: $x(t) = x_p(t) + x_c(t)$ x_p özel çözüm (zorlanmış) x_c tamamlayıcı çözüm (doğal)

ÖRNEK $v(t)$ GERİLİM CEVABINI BELİRLEYİN

$$R = 2\Omega, L = 5H, C = \frac{1}{5}F$$

$$i_L(0) = -1A, v_C(0) = 4V$$

**ADIM 1: MODELİ ELDE EDİN**

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L} \int_0^t v(x) dx + i_L(0) + C \frac{dv}{dt} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0$$

ADIM 2: KARAKTERİSTİK DENKLEM**KARAKTERİSTİK DENKLEM**

$$s^2 + 2.5s + 1 = 0 \quad \Rightarrow \omega_n = 1; \zeta = 1.25$$

ADIM 3: KÖKLERİ BULUN

$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

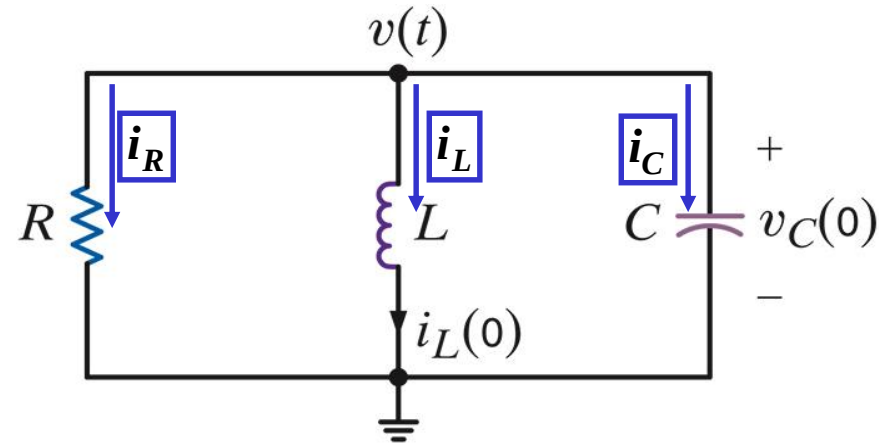
ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN

$$v(t) = K_1 e^{-2t} + K_2 e^{-0.5t}$$

ÖRNEK -devam **$v(t)$ GERİLİM CEVABINI BELİRLEYİN**

$$R = 2\Omega, L = 5H, C = \frac{1}{5}F$$

$$i_L(0) = -1A, v_C(0) = 4V$$

**ADIM 5: SABİTLERİ BULUN**

Sabitleri belirlemek için
ihtiyaç duyduğlarımız;

$$v(0+); \frac{dv}{dt}(0+)$$

$$v(0+) = v_C(0+) = v_C(0) = 4V$$

Eğer verilmemişse $v_C(0)$ ve $i_L(0)$ bulunur

 $t=(0+)$ ' DA DEVREYİ ANALİZ EDİN

$t = (0+)$ ' DA KAK UYGULA

$$\frac{v_C(0+)}{R} + i_L(0+) + C \frac{dv}{dt}(0+) = 0$$

$$\frac{4}{2} - 1 + \frac{1}{5} \frac{dv}{dt}(0+) = 0$$

$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} + \frac{1}{(1/5)} = -5$$

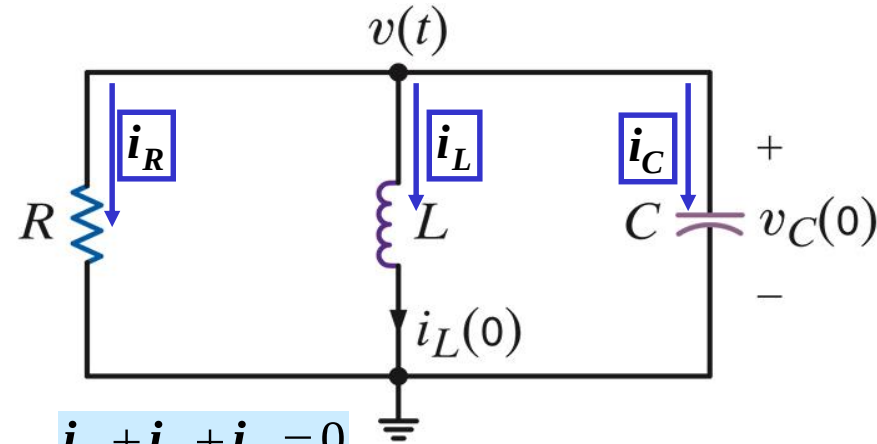
$$\left. \begin{array}{l} K_1 + K_2 = 4 \\ -2K_1 - 0.5K_2 = -5 \end{array} \right\} \Rightarrow K_1 = 2; K_2 = 2$$

$$v(t) = 2e^{-2t} + 2e^{-0.5t}; t > 0$$

ÖRNEK -özet **$v(t)$ GERİLİM CEVABINI BELİRLEYİN**

$$R = 2\Omega, L = 5H, C = \frac{1}{5}F$$

$$i_L(0) = -1A, v_C(0) = 4V$$



$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L} \int_0^t v(x) dx + i_L(0) + C \frac{dv}{dt} = 0$$

**ADIM 1
MODEL**

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0$$

KARAKTERİSTİK DENKLEM**ADIM 2**

$$s^2 + 2.5s + 1 = 0 \Rightarrow \omega_n = 1; \zeta = 1.5$$

$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

**ADIM 3
KÖKLER**

$$v(t) = K_1 e^{-2t} + K_2 e^{-0.5t}$$

**ADIM 4
ÇÖZÜMÜN
BİÇİMİ****ADIM 5:
SABİTLERİ
BULUN****Sabitleri belirlemek için ihtiyaç duyduğlarımız**

$$v(0+); \frac{dv}{dt}(0+)$$

Verilmemiş se $v_C(0)$ ve $i_L(0)$ bulunur

$$v(0+) = v_C(0+) = v_C(0) = 4V$$

 $t = (0+)$ 'DA KAK UYGULA

$$\frac{v_C(0+)}{R} + i_L(0+) + C \frac{dv}{dt}(0+) = 0$$

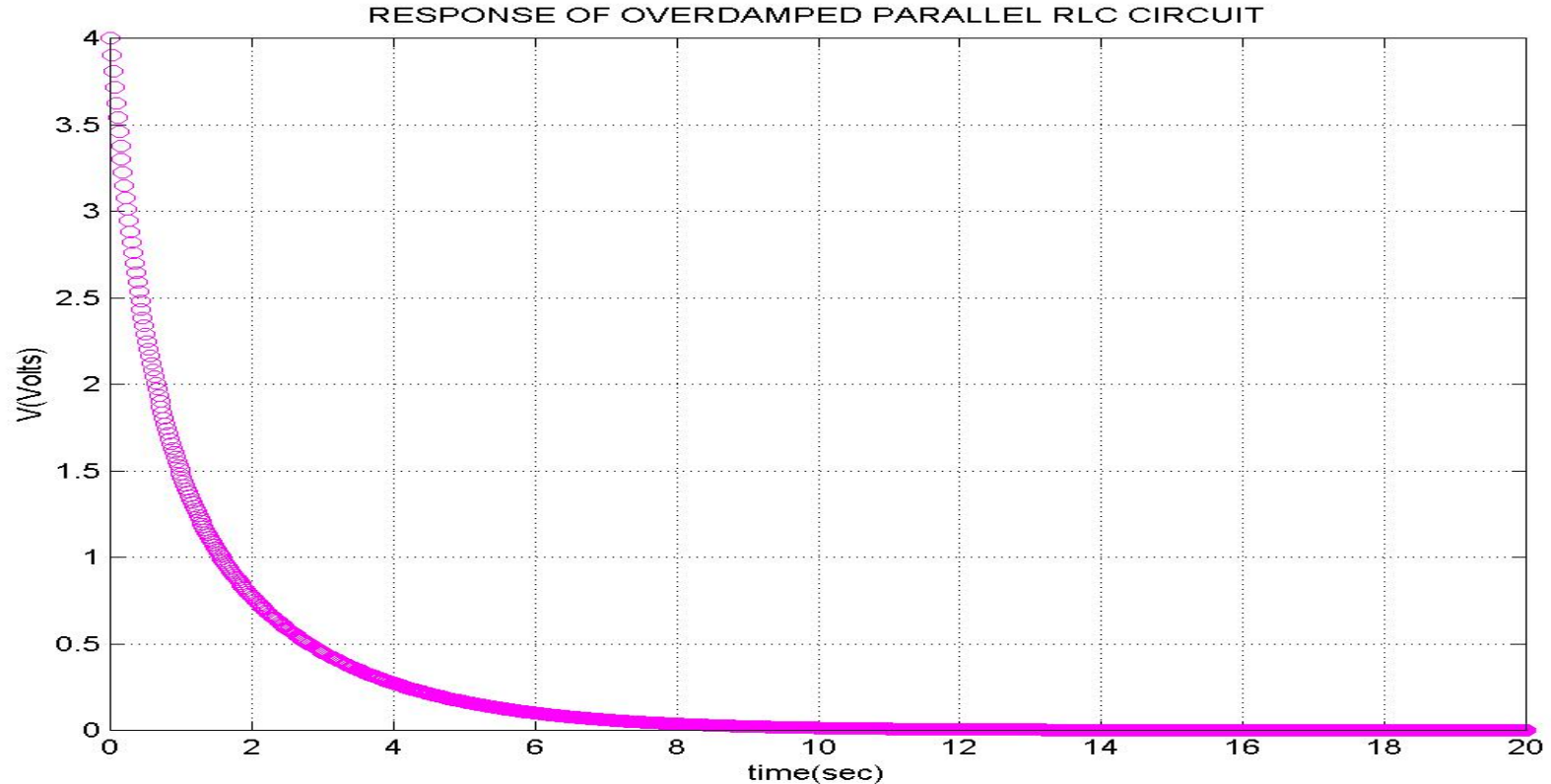
$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} - \frac{(-1)}{(1/5)} = -5$$

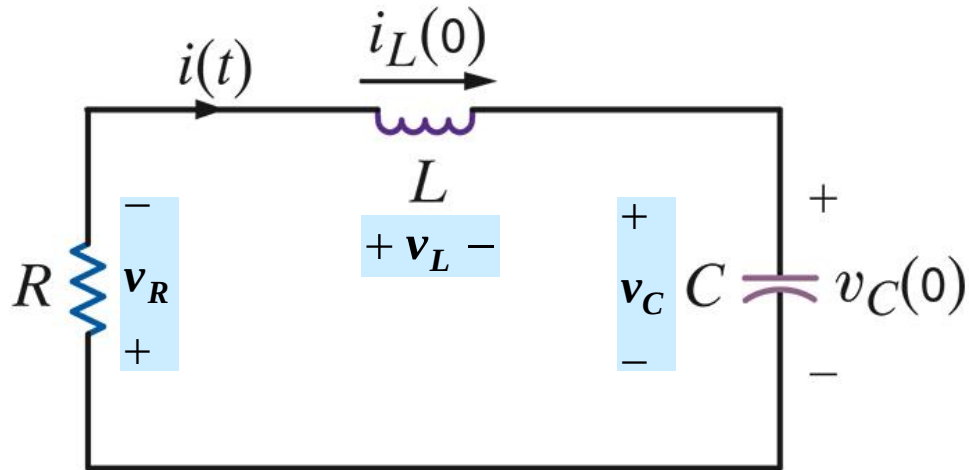
**t=0+'DA
ANALİZ ET**

$$\left. \begin{array}{l} K_1 + K_2 = 4 \\ -2K_1 - 0.5K_2 = -5 \end{array} \right\} \Rightarrow K_1 = 2; K_2 = 2$$

$$v(t) = 2e^{-2t} + 2e^{-0.5t}; t > 0$$

```
%script6p7.m  
%plots the response in Example 6.7  
%v(t)=2exp(-2t)+2exp(-0.5t); t>0  
t=linspace(0,20,1000);  
v=2*exp(-2*t)+2*exp(-0.5*t);  
plot(t,v,'mo'), grid, xlabel('time(sec)'), ylabel('V(Volts)')  
title('RESPONSE OF OVERDAMPED PARALLEL RLC CIRCUIT')
```



ÖRNEK **$i(t)$ AKIM CEVABINI BELİRLEYİN**

$$v_R + v_L + v_C = 0$$

$$Ri(t) + L \frac{di}{dt}(t) + \frac{1}{C} \int_0^t i(x) dx + v_C(0) = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt}(t) + \frac{1}{LC} i(t) = 0$$

$$\frac{d^2 i}{dt^2} + 6 \frac{di}{dt}(t) + 25 i(t) = 0$$

model

$$R = 6\Omega, L = 1H, C = 0.04F$$

$$i_L(0) = 4A; v_C(0) = -4V$$

ADIM 1: MODELİ ELDE EDİN**ADIM 2: KARAKTERİSTİK DENKLEM****ADIM 3: KÖKLERİ BULUN****ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN****ADIM 5: SABİTLERİ BULUN****Karakteristik Denklem :**

$$s^2 + 6s + 25 = 0$$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5$$

$$2\zeta\omega_n = 6 \Rightarrow \zeta = 0.6$$

$$\text{kökler : } s = \frac{-6 \pm \sqrt{36 - 100}}{2} = -3 \pm j4 \omega_d$$

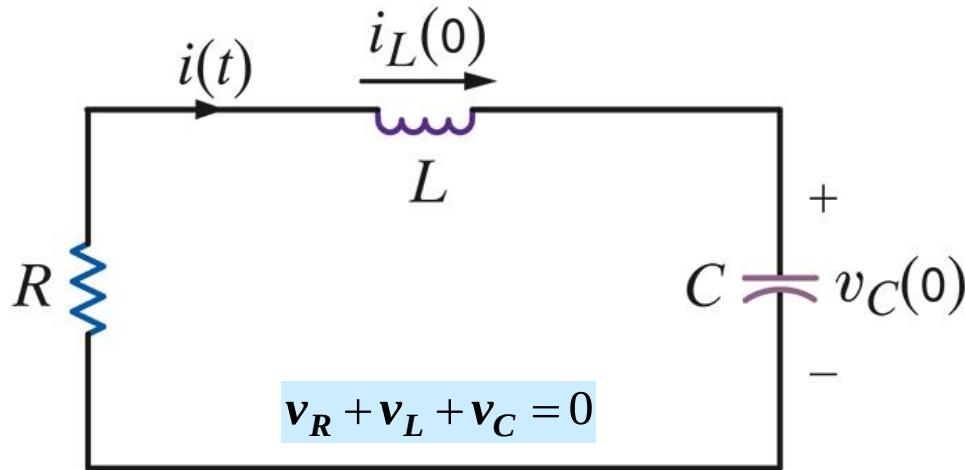
Çözümün biçimi:

$$i(t) = e^{-3t} (A_1 \cos 4t + A_2 \sin 4t)$$

ÖRNEK -devam **$i(t)$ AKIM CEVABINI BELİRLEYİN**

$$R = 6\Omega, L = 1H, C = 0.04F$$

$$i_L(0) = 4A; v_C(0) = -4V$$

**ADIM 1: MODELİ ELDE EDİN✓****ADIM 2: KARAKTERİSTİK DENKLEM✓****ADIM 3: KÖKLERİ BULUN✓****ADIM 4: ÇÖZÜM BİÇİMİNİ YAZIN✓****ADIM 5: SABİTLERİ BULUN**

$$i(t) = e^{-3t} (A_1 \cos 4t + A_2 \sin 4t)$$

$$i(0) = i_L(0) = 4A \Rightarrow 4 = (A_1 + 0) \Rightarrow A_1 = 4$$

$$\frac{di}{dt}(0+)'yi \text{ hesaplamak için } v_L(t) = L \frac{di}{dt}(t)$$

 **$t = 0$ 'da anahtarlama veya süreksizlik yok.
 $t = 0$ veya $t = (0+)$ 'yı kullanın**

$$L \frac{di}{dt}(0) = -Ri(0) - v_C(0)$$

$$\frac{di}{dt}(0+) = -6 \times 4 - (-4) = -20$$

$$\frac{di}{dt}(0+) = -20$$

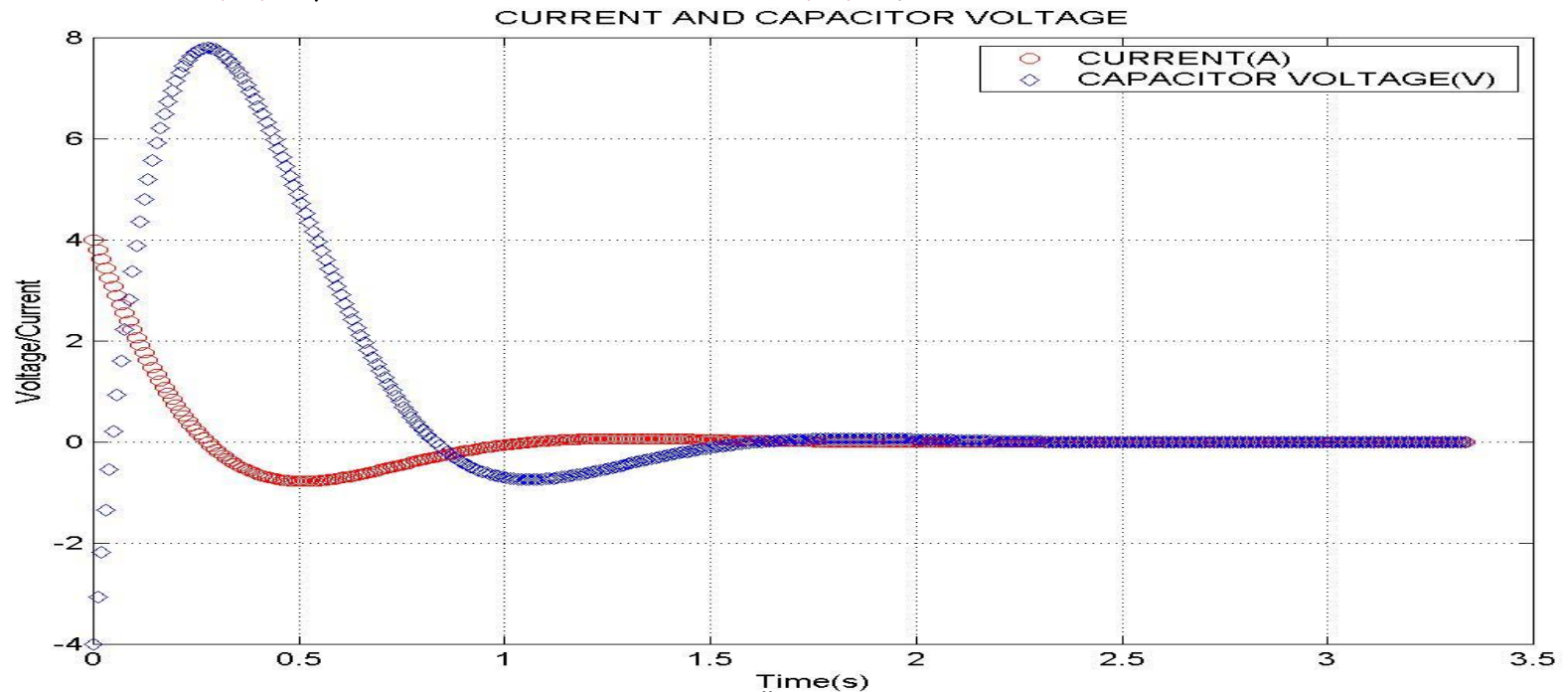
$$t = 0' da: -20 = -3A_1 + 4A_2$$

$$-20 = -3 \times 4 + 4A_2 \Rightarrow A_2 = -2$$

$$i(t) = e^{-3t} (4 \cos 4t - 2 \sin 4t) [A]; t > 0$$

CEVABI GÖRSELLEŞTİRMEK İÇİN MATLAB KULLANMA

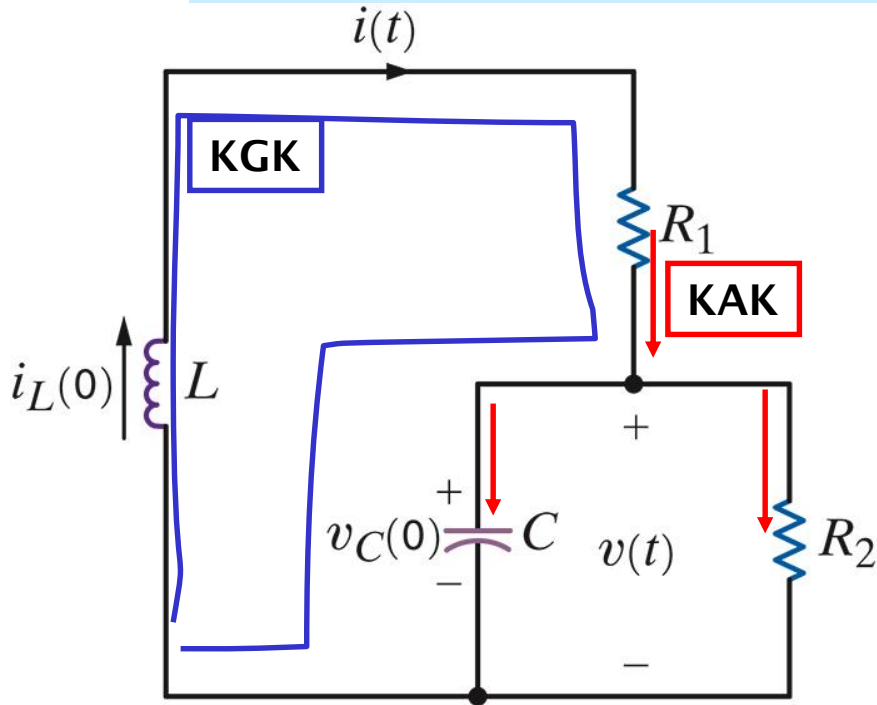
```
%script6p8.m
%displays the function  $i(t)=\exp(-3t)(4\cos(4t)-2\sin(4t))$ 
% and the function  $v_c(t)=\exp(-3t)(-4\cos(4t)+22\sin(4t))$ 
% use a simple algorithm to estimate display time
tau=1/3;
tend=10*tau;
t=linspace(0,tend,350);
it=exp(-3*t).*(4*cos(4*t)-2*sin(4*t));
vc=exp(-3*t).*(-4*cos(4*t)+22*sin(4*t));
plot(t,it,'ro',t,vc,'bd'),grid,xlabel('Time(s)'),ylabel('Voltage/Current')
title('CURRENT AND CAPACITOR VOLTAGE')
legend('CURRENT(A)', 'CAPACITOR VOLTAGE(V)')
```



ÖRNEK **$v(t)$ GERİLİM CEVABINI BELİRLEYİN**

$$R_1 = 10\Omega, R_2 = 8\Omega, C = \frac{1}{8}F, L = 2H$$

$$v_C(0) = 1V, i_L(0) = 0.5A$$



$$L \frac{di}{dt}(t) + R_1 i(t) + v(t) = 0$$

$$i(t) = \frac{v(t)}{R_2} + C \frac{dv}{dt}(t)$$

$$L \left(\frac{1}{R_2} \frac{dv}{dt}(t) + C \frac{d^2v}{dt^2} \right) + R_1 \left(\frac{v(t)}{R_2} + C \frac{dv}{dt}(t) \right) + v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + \left(\frac{1}{R_2 C} + \frac{R_1}{L} \right) \frac{dv}{dt}(t) + \frac{R_1 + R_2}{R_2 L C} v(t) = 0$$

**Değerler yerine yazıldığında
DEVRE MODELİ**

$$\frac{d^2v}{dt^2}(t) + 6 \frac{dv}{dt}(t) + 9v(t) = 0$$

KARAKTERİSTİK DENKLEM

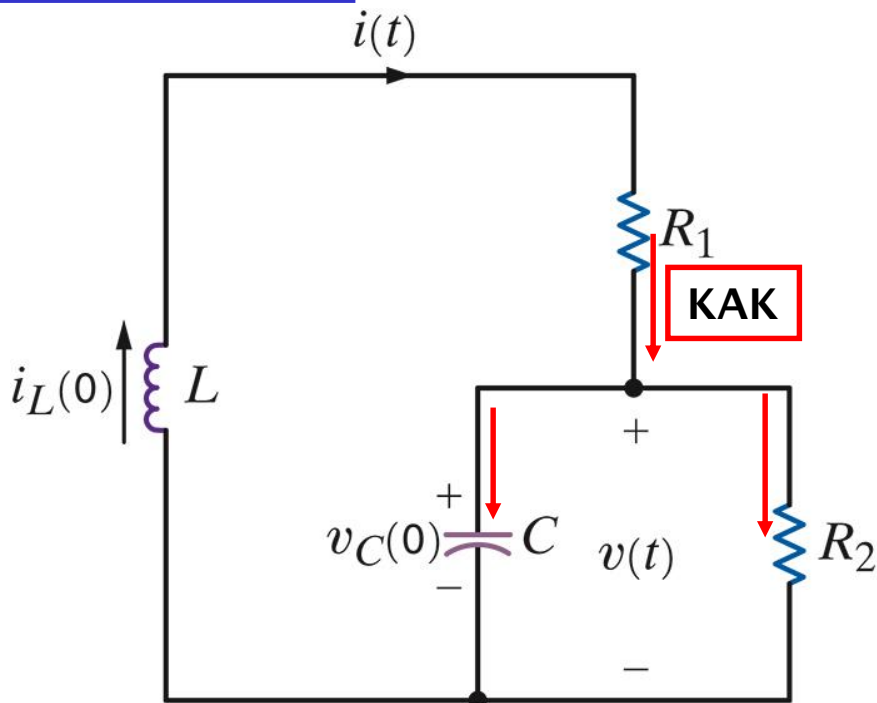
$$s^2 + 6s + 9 = 0$$

$$\omega_n = 3, 2\zeta\omega_n = 6 \Rightarrow \zeta = 1$$

ÖRNEK -devam **$v(t)$ GERİLİM CEVABINI BELİRLEYİN**

$$R_1 = 10\Omega, R_2 = 8\Omega, C = \frac{1}{8}F, L = 2H$$

$$v_C(0) = 1V, i_L(0) = 0.5A$$

**SABİTLER**

$$v(0+) = v_C(0+) = 1V$$

$$v(0) = 1 = B_1$$

**$t = 0$ 'da anahtarlama veya süreksizlik yok.
 $t = 0$ veya $t = (0+)$ 'yı kullanın**

KÖKLER

$$s^2 + 6s + 9 = 0 = (s + 3)^2$$

$$s_1 = s_2 = -3$$

ÇÖZÜMÜN BİÇİMİ

$$v(t) = e^{-3t}(B_1 + B_2 t)$$

$t = (0+)$ 'da KAK

$$i(0) = i_L(0) = \frac{v(0)}{R_2} + C \frac{dv}{dt}(0) \Rightarrow \frac{dv}{dt}(0) = 3$$

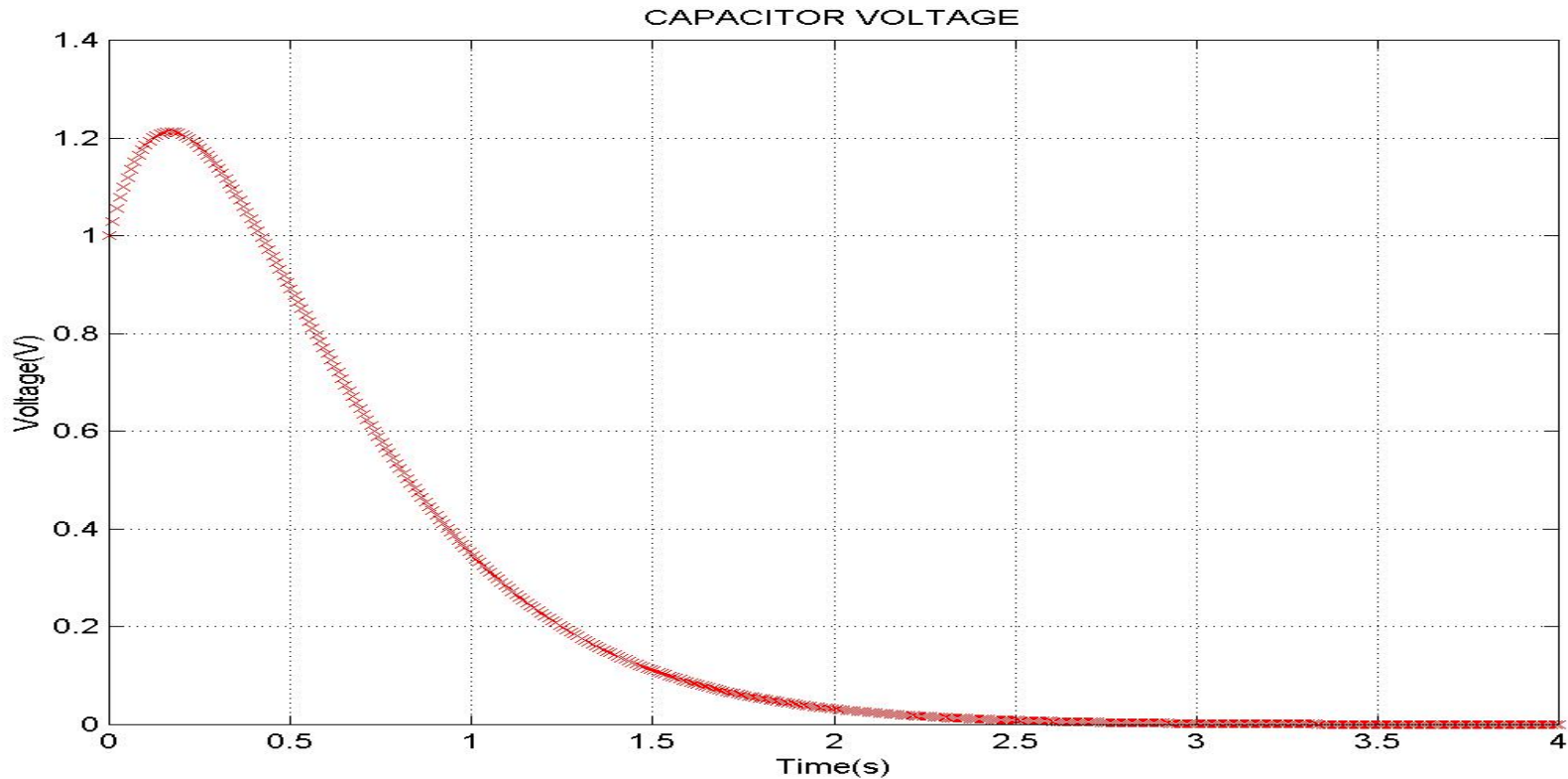
$$\frac{dv}{dt}(0) = -3e^{-3t}(B_1 + B_2 t) + B_2 e^{-3t}$$

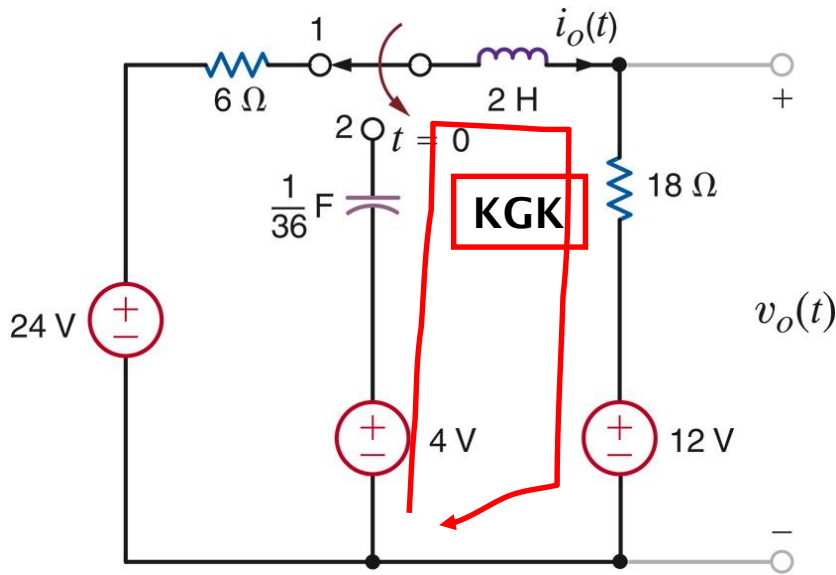
$$\frac{dv}{dt}(0) = -3B_1 + B_2 = 3 \Rightarrow B_2 = 6$$

$$v(t) = e^{-3t}(1 + 6t); t > 0$$

CEVABI GÖRSELLEŞTİRMEK İÇİN MATLAB KULLANMA

```
%script6p9.m  
%displays the function  $v(t)=\exp(-3t)(1+6t)$   
tau=1/3;  
tend=ceil(10*tau);  
t=linspace(0,tend,400);  
vt=exp(-3*t).*(1+6*t);  
plot(t,vt,'rx'),grid, xlabel('Time(s)'), ylabel('Voltage(V)')  
title('CAPACITOR VOLTAGE')
```



ÖRNEK $t > 0$ için $i_0(t)$ ve $v_o(t)$ 'yi bulun**DEVRE MODELİ**

$$-4 + \frac{1}{1/36} \int_0^t i(x) dx + v_C(0) + 2 \frac{di}{dt}(t) + 18i(t) + 12 = 0$$

$$\frac{d^2 i}{dt^2}(t) + 9 \frac{di}{dt}(t) + 18i(t) = 0$$

$$v_o(t) = 18i_0(t) + 12(V)$$

KARAKTERİSTİK DENKLEM

$$s^2 + 9s + 18 = 0$$

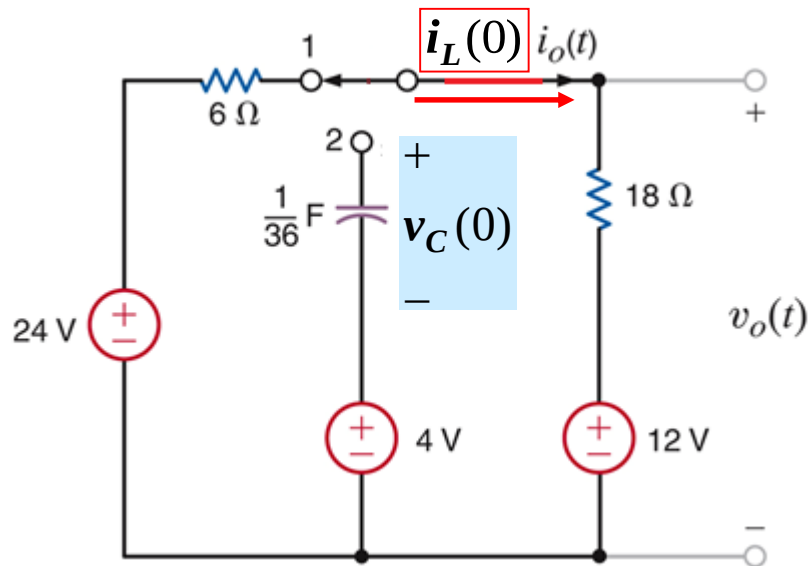
KÖKLER

$$s_1 = -3, s_2 = -6$$

ÇÖZÜMÜN BİÇİMİ

$$i_0(t) = K_1 e^{-3t} + K_2 e^{-6t}; t > 0$$

Başlangıç şartlarını bulmak için $t < 0$ 'da kalıcı durum analizini kullanın



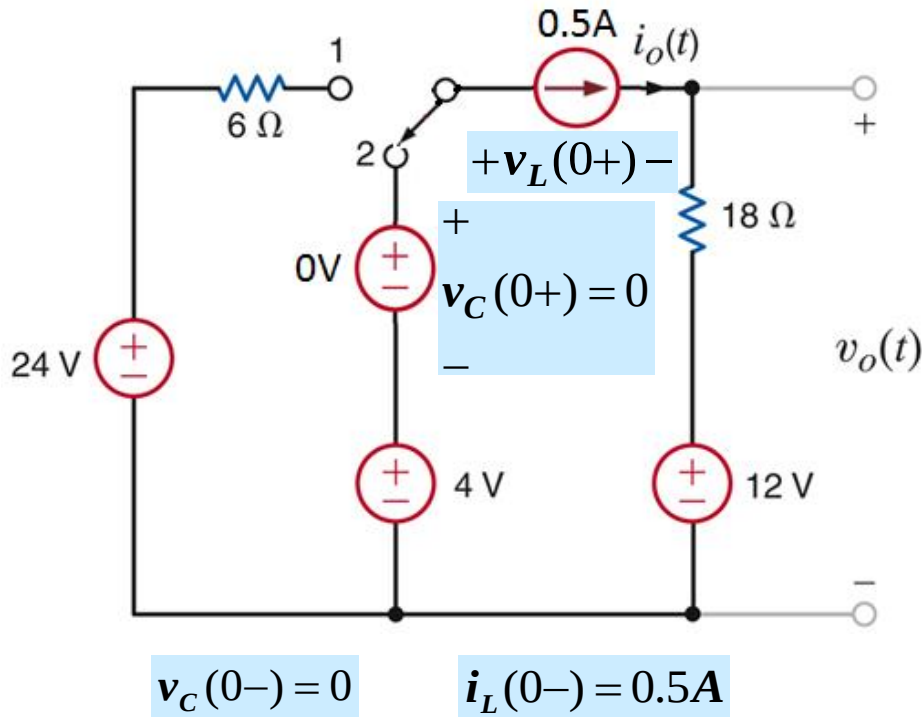
$$v_C(0^-) = 0$$

$$i_L(0^-) = 0.5 \text{ A}$$

ÖRNEK -devam

$t > 0$ için $i_0(t)$ ve $v_o(t)$ 'yi bulun

$t=0^+$ 'da devreyi analiz edin



$$v_o(t) = 18i_0(t) + 12(\text{V})$$

$$i_0(t) = K_1 e^{-3t} + K_2 e^{-6t}; t > 0$$

$$i_0(0+) = i_L(0+) = 0.5(\text{A})$$

$$i_0(0+) = 0.5 = K_1 + K_2$$

$$v_L(0+) = L \frac{di_L}{dt}(0+) = L \frac{di_0}{dt}(0+)$$

$$-4 + 2 \frac{di_L}{dt}(0+) + 18i_L(0+) + 12 = 0$$

$$\frac{di_0}{dt}(0+) = -17/2$$

$$\frac{di_0}{dt} = -3K_1 e^{-3t} - 6K_2 e^{-6t}; t = 0+$$

$$\frac{di_0}{dt}(0+) = -17/2 = -3K_1 - 6K_2$$

$$-17/2 = -3K_1 - 6K_2$$

$$0.5 = K_1 + K_2$$

$$K_1 = -\frac{11}{6}; \quad K_2 = \frac{14}{6}$$

$$i_0(t) = -\frac{11}{6} e^{-3t} + \frac{14}{6} e^{-6t}; t > 0$$

$$v_o(t) = 12 + 18i_0(t); t > 0$$